

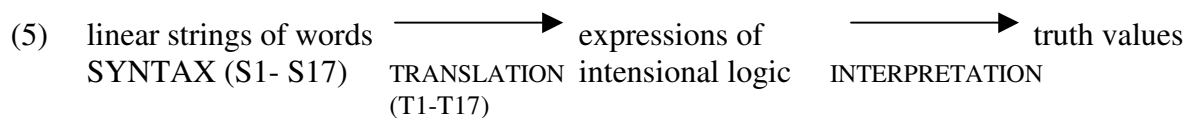
## Montague (1973): The Proper Treatment of Quantification in Ordinary English (PTQ)

### A. Central data to be accounted for

- i. Proper names and QNPs in subject position: [<sub>S</sub> QNP VP]
- (1) a. Bill walks.  
 b. every / a man walks.
- ii. QNPs in object position: [NP [<sub>VP</sub> V QNP]
- (2) John finds a unicorn
- iii. scope ambiguity in non-intensional contexts: [<sub>S</sub> QNP [<sub>VP</sub> V QNP]
- (3) a woman loves every man.
- iv. scope ambiguity in intensional contexts (de re / de dicto) [<sub>S</sub> NP [<sub>VP</sub> V intens QNP]
- (4) John seeks a unicorn
- v. pronouns in intensional environments and sentences about individual concepts:
- (5) a. John wishes to catch a fish and eat it.  
 b. The temperature is 90, but it is rising.

### B. The Grammatical System

- The grammatical system of Montague (1973) is formalized within the framework of categorial grammar. The categorial specification of the basic expressions of the language governs their combinatory possibilities with other expressions. Syntactic rules define strings of words (SYNTAX), which together form the set of grammatical expressions of (a fragment of) English. For any syntactic rule there is a corresponding translation rule that translates its output into an expression of intentional logic, viz. the translation rules (T1- T19). The output of the translation rules is then interpreted in an intensional semantic framework, i.e. relative to possible worlds and times.



NB: The intermediate step of translating into intentional logic is more perspicuous, but not an absolute necessity. In other work, Montague employs direct semantic interpretation of syntactic strings.

- *Basic expressions (inductive definition):*

- (6) a. intransitive verbs (IV): expressions that map an entity expression onto a truth-value expression (= a sentence): t/e  
 b. terms (T): expressions that map an IV onto a truth-value expression: t/IV  
 c. transitive verbs (TV): expressions that map a term onto an intransitive verb: IV/T  
 d. IV-modifying adverbs (IAV): IV/IV  
 e. common nouns (CN): expressions that map an entity expression onto a truth-value expression: t/e

Note: The syntactic differences between IV and CN are captured by the notational difference **t/e** vs. **t/e**.

- *Basic expressions (examples)*

- (7)  $B_{IV} = \{\text{run, walk, snore, ...}\}$   
 $B_T = \{\text{John, Mary, Bill, ninety, Hamburg, he}_0, \text{he}_1, \dots\}$   
 $B_{TV} = \{\text{find, love, eat, seek...}\}$   
 $B_{IAV} = \{\text{slowly, carefully, ...}\}$   
 $B_{CN} = \{\text{man, woman, unicorn, price, fish, ...}\}$

The most striking feature of this system is that all NPs, including proper names and pronouns are terms of categorial type  $t/IV \rightarrow$  **All nominal expressions are generalized quantifiers**

$\rightarrow$  on this treatment, the proper name *Peter* denotes the set of properties that Peter has (including the property of being self-identical with Peter)

### C. Syntactic Rules

- The syntactic rules S1-S17 govern the combination of basic and complex expressions of various types, and thus derive the and only the *grammatical sequences* of the fragment of English.

The rules specify the category of the input expressions (to be combined), the category of the output expressions, and the syntactic form of the output expression.

- (8) if  $\alpha, \beta, \dots \in P_{CAT}$ , then  $F(\alpha, \beta, \dots) \in P_{CAT}$ , where  $F(\alpha, \beta, \dots) =$

- *Basic (syncategorematic) rules*

S2: derives terms (T) from a common noun (CN) and the determiners *every, the, a(n)*

- (9) a.  $F_0(\text{man}) = \text{every man}$   
 b.  $F_1(\text{man}) = \text{the man}$   
 c.  $F_2(\text{man}) = \text{a man}$

$\rightarrow$  unlike in later frameworks, complex DPs are formed by a syncategorematic rule

S3: forms a sequence of CN plus relative clause from a CN and a sentence containing a (relativized) pronoun with index  $n$ .

- (10)  $F_{3,n}(\text{man, he}_n \text{ caught a fish}) = \text{man such that he caught a fish}$

- *Rules of functional application*

S4: functional application of nominal term ( $t/IV$ ) to intransitive predicate (IV) to yield a truth-value expression:  $t/IV + IV \rightarrow t$

- (11)  $F_4(\text{bill, walk}) = \text{bill walks}$

$\rightarrow$  syntactically, the subject term takes the intransitive predicate as argument

S5: functional application of transitive verb (IV/T) to nominal term (T) to yield an IV:  
 $IV/T + T \rightarrow IV$

- (11)  $F_5(\text{catch, a unicorn}) = \text{catch a unicorn}$

S7 is the rule for transitive verbs that take sentential complements ( $IV/t + t \rightarrow IV$ )

S8 – S10 are the rules for adverbial modification

S11-S13 are the rules for conjunction and disjunction at the sentential level (= t) (S11), the VP-level (= IV) (S12), and the DP-level (= term) (S13)

Note: Conjunction is not defined for the level of terms, such that **Max and Bill arrived** cannot be analysed as  $[_T \text{Max and Bill}] [_{IV} \text{arrived}]$ .

• **Rules of quantification:**

The rules of quantification are necessary in order to account for instances of variable binding by a QNP (*Every man<sub>1</sub> admitted that he<sub>1</sub> had committed a tax offense*) and for cases where a QNP extends its syntactic and semantic scope beyond its surface position. Rules S14 – S16 all have the same general scheme:

A term expression  $\alpha$  combines with an expression that contains one or more pronouns with index n, where  $\alpha$  replaces the first occurrence of the pronoun while the other n-indexed pronouns are replaced by ordinary pronominal forms

S14 combines terms with pronoun-containing sentences to yield a quantified sentence:

- (12) a.  $F_{10,n}(\text{every man, he}_n \text{ walks}) = \text{every man walks}$   
 b.  $F_{10,n}(\text{every man, I found him}_n) = \text{I found every man}$

S15 and S16 combine terms with pronoun-containing VPs and NPs, respectively.

- (13) a.  $F_{10,n}(\text{every student, saw him}_n) = \text{saw every student}$   
 b.  $F_{10,n}(\text{every student, brother of him}_n) = \text{brother of every student}$

S17 syncategorematically introduces negation and tense operators

→ there are no basic syntactic expressions corresponding to Neg, T etc.

**D. Sample Derivation (p.253)**

(14) every man loves a woman such that she loves him

- S1: love  $\in P_{IV/T}$ , he<sub>0</sub>  $\in P_T$   
 S5: loves him<sub>0</sub>  $\in P_{IV}$  (FA: verb<sub>tr.</sub> + object NP)  
 S1+S4: he<sub>1</sub> loves him<sub>0</sub>  $\in P_{IV}$  (FA: subject NP + VP)  
 S1+S3: woman such that she loves him<sub>0</sub>  $\in P_{CN}$  (relative clause)  
 S2: a woman such that she loves him<sub>0</sub>  $\in P_T$  (a + CN)  
 S1+S5: loves a woman such that she loves him<sub>0</sub>  $\in P_{IV}$  (FA: verb<sub>tr.</sub> + object NP)  
 S1+S4: he<sub>0</sub> loves a woman such that she loves him<sub>0</sub>  $\in P_t$  (FA: subject NP + VP)  
 S1+S2: every man him<sub>0</sub>  $\in P_T$  (every + CN)  
 S14: every man loves a woman such that she loves him  $\in P_T$

### E. The relation of syntax and semantics: Ambiguities

- *Structural Ambiguities*

The resulting syntactic system gives rise to structural ambiguities (one reason being that there is more than one way to combine a term with other material to form sentences or IVs, plus the fact that the system does not discriminate between proper names and QNPs, which are all terms.

p.254: "It can be shown that every declarative sentence of our fragment has infinitely many analysis trees."

(15) The derivation of *Peter walks / Every man walks*

Both sentences can be derived in two ways.

S1: walk  $\in P_{IV}$ , Peter  $\in P_T$ , S2: every man  $\in P_T$

i. S4: Peter walks  $\in P_t$ , Every man walks  $\in P_t$

ii. S4: he<sub>1</sub> walks  $\in P_t$

S14: Peter /Every man + he<sub>1</sub> walks  $\in P_t \rightarrow$  Peter walks  $\in P_t$  / Every man walks  $\in P_t$

$\rightarrow$  Both analyses (or derivations) lead to the same semantic result

- *Semantic Ambiguities*

In certain cases, different analyses lead to different semantic interpretations:

p.255: "our fragment admits genuinely (that is, semantically) ambiguous sentences"

(16) one linear sequence of words  $\rightarrow$  more than one interpretation

(17) John seeks a unicorn:

i. object NP combines with transitive verb by S5: [John [seeks [ a unicorn]]]

$\rightarrow$  *de dicto*: John seeks an entity with the property of being a unicorn

ii. object NP is quantified in by (S14):

a unicorn + [John [seeks [ him]]  $\rightarrow$  John seeks a unicorn

$\rightarrow$  *de re*: There is an entity with the property of being a unicorn such that John seeks it

$\rightarrow$  the *de re*-reading is the result of translation rule T14, which accompanies S14 (recall that it is not the output of a syntactic rule that is translated, but that the syntactic rules come with corresponding translation rules)

- A precursor of LF and quantifier raising!

p.255: "If it were desired to construct a corresponding unambiguous language, it would be convenient to take the analysis trees themselves as the expressions of that language."
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### F. Intensional Logic

Before giving the translation rules T1- T 17, Montague introduces the general system of intensional logic that he uses as the basis for semantic evaluation (= interpretation). The key feature of intensional systems is that the extensional denotation of a linguistic expressions is a function of a possible world  $I \in I$ , and a moment of time  $j \in J$ , or rather an *index*  $\langle i, j \rangle \in I \times J$ . The semantic type of indices is  $\langle s \rangle$ .

The intension of an expression  $\alpha$  is its conceptual meaning, i.e. a function from indices into an object of type  $e$ ,  $t$ , or any combination thereof.

Applying the intension to an index yields the extension:  $EXT(\alpha) = [INT(\alpha)](\langle i, j \rangle)$

In intensional logic, one can switch between the extension and intension by means of the two operators ' $\wedge$ ' and ' $\vee$ ':

The operator ' $\wedge$ ' intensionalizes a meaningful expression of type  $\langle a \rangle$  to  $\langle s, a \rangle$

The operator ' $\vee$ ' extensionalizes a meaningful expression of type  $\langle s, a \rangle$  to  $\langle a \rangle$

The assumption of intensional types  $\langle s, \dots \rangle$  enlarges the set of relevant semantic classes of meaningful expressions (p.259):

- (18) a. next to set-denoting expressions of type  $\langle a, t \rangle$ , there are property-denoting expressions of type  $\langle s, \langle a, t \rangle \rangle$
- b. next to relation-denoting expressions of type  $\langle a, \langle b, t \rangle \rangle$ , there are expressions denoting relation-in-intensions of type  $\langle s, \langle a, \langle b, t \rangle \rangle \rangle$
- c. next to truth-value denoting expressions of type  $\langle t \rangle$ , there are proposition-denoting expressions of type  $\langle s, t \rangle$
- d. next to individual-denoting expression of type  $\langle e \rangle$ , there are expressions denoting individual concepts of type  $\langle s, e \rangle$

→ another feature of Montague's intensional system:

The negative operator  $\neg$ , tense (W,H) and modality operators apply to full formulas. There is no counterpart to these expressions in the syntactic component. The operators are introduced by rule T17.

- Restricting the intensional system:

As it is, the intensional system defined by means of the translation rules in T1-T17 is much more expressive than the natural language English:

p.263: "Not all interpretations of intensional logic, however, would be reasonable candidates for interpretations of English"

→ Restriction of the intensional system by means of meaning postulates (1)-(9) on p.263:

- (19) proper names are rigid designators (denote the same entity across possible worlds) (1)
- most intransitive expressions are extensional wrt the subject argument (3)
- most transitive expressions are extensional wrt subject and object argument (4)
- the transitive expressions *seek* and *conceive* (and verbs taking sentential complements such as *believe*, *assert*) are extensional wrt to the subject argument only (5), (6)
- the verb *seek* is semantically complex and analysed as *try to find* (9)

**G. Translating English sentences** (abstracting away from intensionality where possible)

- *Basic cases*

(20) a. Bill walks (S4, T4)  $\rightarrow$   $[\lambda P. P(b)] (\lambda x. \text{walk}'(x)) = \mathbf{walk}'(b)$

b. Bill walks (S14, T14)  $\rightarrow$   $[\lambda P. P(b)] (\lambda x_n. [\lambda P. P(x_n)](\lambda z. \text{walk}'(z)))$   
 $\Leftrightarrow$   $[\lambda P. P(b)] (\lambda x_n. \text{walk}'(x_n))$   
 $\Leftrightarrow$   $\mathbf{walk}'(b)$

(21) a. Every man walks (S4, T4)  $\rightarrow$   $[\lambda P. \forall u[\text{man}'(u) \rightarrow P(u)] (\lambda x. \text{walk}'(x))$   
 $\Leftrightarrow$   $\forall u [\mathbf{man}'(u) \rightarrow \mathbf{walk}'(u)]$

b. Every man walks (S14, T14)  
 $\rightarrow$   $[\lambda P. \forall u[\text{man}'(u) \rightarrow P(u)] (\lambda x_n. [\lambda P. P(x_n)](\lambda z. \text{walk}'(z)))$   
 $\Leftrightarrow$   $[\lambda P. \forall u[\text{man}'(u) \rightarrow P(u)] (\lambda x_n. \text{walk}'(x_n))$   
 $\Leftrightarrow$   $\forall u [\mathbf{man}'(u) \rightarrow \mathbf{walk}'(u)]$

(22) a. John finds a unicorn.  
 finds a unicorn (S5, T5)

$\rightarrow$   $[\lambda \wp \in D_{\langle et, t \rangle}. \lambda x. \wp([\lambda y. [\lambda z. \text{find}'(z, y)](x)])](\lambda P. \exists u[\text{unicorn}'(u) \wedge P(u)])$

note: For T5 to apply, the meaning of the transitive verb must be type-shifted to  $\langle\langle et, t \rangle, \langle et \rangle\rangle$  !

$\Leftrightarrow$   $\lambda x. [\lambda P. \exists u[\text{unicorn}'(u) \wedge P(u)]] ([\lambda y. \text{find}'(x, y)])$   
 $\Leftrightarrow$   $\lambda x. \exists u[\text{unicorn}'(u) \wedge \text{find}'(x, u)]$

Application of  $[\lambda P. P(j)]$  with S4/T4 gives  $\exists u[\mathbf{unicorn}'(u) \wedge \mathbf{find}'(j, u)]$

- Scope ambiguities in non-intensional contexts

(23) a woman loves every man.

- i. SUBJ > OBJ: there is a woman that loves all men
  - ii. OBJ > SUBJ: for every man there is a woman that loves him
- a. SUBJ > OBJ

derivation of the meaning for *loves every man* as in (22) (S5/ T5):

$\lambda x. \forall u[\text{man}'(u) \rightarrow \text{love}'(x, u)]$

combination with subject QP via S4/T4:

$[\lambda P. \exists v [\text{woman}'(v) \wedge P(v)]] (\lambda x. \forall u[\text{man}'(u) \rightarrow \text{love}'(x, u)])$

$\Leftrightarrow \exists v [\mathbf{woman}'(v) \wedge \forall u[\mathbf{man}'(u) \rightarrow \mathbf{love}'(v)(u) ]]$

b. OBJ > SUBJ

derivation of the meaning for *loves him<sub>0</sub>* (as in (22)) (S5/ T5):

$\lambda x. \text{love}'(x)(x_0)$

combination with subject QP via S4/T4:

$[\lambda P. \exists v [\text{woman}'(v) \wedge P(v)]] (\lambda x. \text{love}'(x, x_0))$

$\Leftrightarrow \exists v [\text{woman}'(v) \wedge \text{love}'(v, x_0)]$

Quantifying In (as in (21b)) (S14/T14):

*every man + a woman loves him<sub>0</sub>*  $\rightarrow$

$[\lambda P. \forall u[\text{man}'(u) \rightarrow P(u)] (\lambda x_0. \exists v [\text{woman}'(v) \wedge \text{love}'(v, x_0)])$

$\Leftrightarrow \forall u[\text{man}'(u) \rightarrow \exists v [\text{woman}'(v) \wedge \text{love}'(v, u)]]$

- de re / de dicto ambiguities

(24) John seeks a unicorn

de dicto: *seeks* combines with *a unicorn* via S5/T5, and then with the subject via S4/T4:

$\rightarrow \text{seek}'(j, \lambda P. \exists u [\text{unicorn}'(u) \wedge P(u)])$

de re: *seeks* combines with a pronoun *him<sub>0</sub>* via S5/T5, then with the subject *john* via S4/T4, and finally with the object QP *a unicorn* via S14/T14:

$[\lambda P. \exists u [\text{unicorn}'(u) \wedge P(u)] (\lambda x_0. \text{seek}'(j)(x_0))$

$\Leftrightarrow \exists u [\text{unicorn}'(u) \wedge \text{seek}'(j)(u)]$

## H. Why Intensionality ?

(25) a. The temperature is 90.

b. The temperature is rising.

c.  $// \rightarrow //$  90 is rising.

$\rightarrow$  the non-sequitur of (25c) is unexpected if the DP *the temperature* denotes an individual in (25).

$\rightarrow$  it follows, however, if *the temperature* denotes an individual concept (<s,e>), and the verb *rising* in (25b) applies to such individual concepts.

$\rightarrow$  90 does not denote an individual concept, but an individual (degree). It therefore cannot be substituted for *the temperature* in (25b).