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Research Note

Comparing Two Smoothing Approaches in Estimating Kinematic Parameters

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ABSTRACT

Purpose: We compare two signal smoothing and differentiation approaches: a frequently used approach in the speech community of digital filtering with approximation of derivatives by finite differences and a spline smoothing approach widely used in other fields of human movement science.
Method: In particular, we compare the values of a classic set of kinematic parameters estimated by the two smoothing approaches and assess, via regressions, how well these reconstructed values conform to known laws about relations between the parameters.
Results: Substantially smaller regression errors were observed for the spline smoothing than for the filtering approach.
Conclusion: This result is in broad agreement with reports from other fields of movement science and underpins the superiority of splines also in the domain of speech.

Speech scientists are faced with the problem of reconstructing continuous physiological signals from measurements taken at discrete points in time (e.g., the position of speech articulators as they move in space). Reconstruction of these signals requires a process called smoothing, which aims to recover important patterns in the measurements while leaving out noise. In speech, smoothing is typically implemented by means of digital low-pass filters with small cutoffs to eliminate high-frequency noise (e.g., Abur et al., 2022; Shellikeri et al., 2016; van Lieshout & Neufeld, 2014). Another separate problem in signal reconstruction is that of differentiation. Movement data registration devices typically provide positional information only, but research studies with such data often require information about also the velocity and acceleration of movements (i.e., the first and second derivatives of the positional signal). The problem of computing derivatives from noisy signals is far from trivial, and several decades of research have led to different solutions (see Medved, 2001; Woltring, 1985; Wood, 1982, for exhaustive overviews). An easy-to-implement approach widely used in speech movement science is that of finite differences, which approximates derivatives by differences in a signal's neighboring samples.

In other fields of human movement science, a different approach for physiological signal reconstruction, which relies on smoothing splines (de Boor, 2001; Eubank, 1999; Medved, 2001; Schumaker, 2007; Wahba, 1990), has gained traction. As in the filtering approach, the splines approach ensures that the signal and (at least) its first and second derivatives are reconstructed as continuous curves (in conformity with the smoothness property of biological signals; cf. Harris & Wolpert, 1998; Sejnowski, 1998). However, instead of handling the processes of signal smoothing and signal differentiation separately, the spline smoothing approach addresses both of these problems at once. Concisely summarized, a smoothing spline is a piecewise polynomial approximation of noisy data points for which a single regularization parameter controls the balance between the smoothness of the approximation and the goodness of its fit to the data. Based on works by Lyche et al. (1983) and Craven and Wahba (1978), Woltring (1986) has developed ways to infer the value of the control parameter from a signal's expected noise characteristics to obtain a spline with deviation from the given data by a fixed predicted mean squared error (e.g., the expected error of the data registration device). The splines approach is independent of the underlying sampling process; in particular, it

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does not depend on the sampling rate and the samples can be arbitrarily spaced in time (ibidem).

In measuring kinematics, whereas speech scientists have relied for many years on the digital filtering with approximation of derivatives by a finite differences approach, in nonspeech movement science, the spline smoothing approach has become the norm; see the works of Wood (1982), Vaughan (1982), D'Amico and Ferrigno (1992), Gazzani (1994), Walker (1998), and Epps et al. (2010) for evidence on the superiority of splines in nonspeech movement science. We are not aware of any comparison between these two approaches of signal reconstruction in speech articulatory data. Thus, this research note is devoted to informing the speech community about the relative performance of the two approaches as this information is essential to proper hypothesis testing and reproducibility in the field of speech movement science.

In light of the extensive literature from nonspeech movement science on the superiority of splines, we hypothesize that the splines will outperform the filtering approach also in the domain of speech. To test this hypothesis, we compared the two approaches in signal reconstruction using speech movements registered with electromagnetic articulography (EMA; Gafos & Goldstein, 2012; Rebernik et al., 2021). Specifically, we collected speech movement data in a paradigm designed to elicit articulations of high kinematic variation, a prerequisite for thoroughly assessing potential differences in the reconstruction of the underlying kinematics. The registered EMA data were then processed twice, using the two smoothing approaches under evaluation. From the resulting smooth articulatory signals, we estimated values for the kinematic parameters of movement duration, movement amplitude, peak velocity, and peak acceleration/deceleration. Substantial differences in the estimates of these parameters were observed. To assess which of the two smoothing approaches better capture the kinematic structure of speech, we compared the performance of the approach-specific parameter estimates in light of three well-documented kinematic relations in the speech production literature. Specifically, via regressions, we assessed how well the approach-specific parameter values conform to three known laws about relations between these values. Smaller regression errors were observed for the splines approach. This result demonstrates the superiority of the splines over the filtering and finite-differences approach also in speech.

This research note is structured as follows: in the Method section, we first present our experimental paradigm of repeated syllable production designed to elicit articulatory movements of high kinematic variation. We then describe how the set of kinematic parameters crucial to this work were extracted from the articulatory signals and present the key measure used in this work (relative percentage difference [RPD]) to quantify the degrees of difference between the splines and filtering approaches. In the Results section, we first verify the high kinematic variation achieved by our experimental paradigm. We then quantify the degrees of difference in the approach-specific kinematic parameter estimates and compare their performance in regressions of three established kinematic relations known to exist among them. In the Discussion section, we finally summarize the results and contrast our approach of performance assessment to past accuracy assessments of data registration devices. We furthermore address potential problems of using the spline smoothing approach in existing laboratory protocols. We finish with the Conclusion section.

Materials and Methods

Five native speakers of German (three females, two males) and five native speakers of English (three females, two males) were recruited at the authors' institution to participate in an experiment of repeated syllable production. Participation in the experiment was paid, and written informed consent was obtained from each speaker prior to experimentation. The experiment had been approved by the Ethics Committee of the University of Potsdam, and all experimental procedures were performed in compliance with the approval. The speakers were between 18 and 35 years old (M = 25.6 years; SD = 5.6 years) and without any self-reported past or present speech or hearing problems. During the experiment, speakers were asked to produce sequences of repeated /ka/ or /ta/ syllables in time with an audible metronome. In blocks of four trials, the rate of the metronome was progressively increased across the values of 150, 210, 300, 390, and 480 beats per minute (bpm). The rate of the metronome served as an extrinsic index of the intended rate of syllable production, covering the ranges of slow (down to 150 bpm), normal (around 300 bpm), and fast speech rates (up to 480 bpm); see the works of Gerstenberg et al. (2018), Pellegrino et al. (2003, 2011), and Dellwo and Wagner (2003) for what ranges of rates are considered normal in German and English. Within a trial, whose length was fixed to allow for the production of 30 consecutive syllables, the speakers were permitted to begin and stop articulation at a point of their choice. On average, our speakers produced 24.7 syllables per trial (SD = 1.6 syllables). The experiment begun with the production of /ka/ syllables, across all rates, and went on with /ta/ syllables afterward.

Regarding the two languages of our speakers, German and English, there was no particular reason for selection other than providing a solid empirical basis for our study. In these two languages, the elicited syllables, /ka/ and /ta/, implicate movements of the two major parts of the tongue, and the low vowel /a/ maximizes the excursion from the consonantal constriction to the vowel. In combination with the systematic manipulation of metronome rate, our study design thus allowed collection of a data set with substantial variation in the kinematic parameters of position, velocity, and acceleration (as we demonstrate in the Results section). The presence of high variation in the data is imperative when comparing the performance of smoothing approaches, which attempt to reconstruct these parameter values. This is even more so when the performance of these approaches is evaluated with respect to how well they capture relations among these parameters: the strength of a relation between two (or more) parameters is best brought out when the isolated parameters vary. (In fact, a prerequisite in stating that there is a relation between any two parameters is the presence of considerable variation in their values.)

Data registration was conducted using latest generation EMA (Carstens AG501) providing three-dimensional positional measures of preselected effectors at the highest spatiotemporal resolution available to date (device specification: 0.3 mm root-mean-square error, 1250 Hz sampling rate). Besides a number of rigid reference locations (left and right mastoids, nose bridge) required for head movement correction (using Horn's method of absolute orientation; Horn, 1987), we registered the positional trajectories of the two primary effectors involved in the formation and release of constrictions in /ka/ and /ta/ syllables. These are the tongue body effector, tracked by a sensor placed midsagittally about 5 mm in front of the /k/-closure contact area of the tongue, and the tongue tip effector, tracked by another sensor placed midsagittally about 10 mm away from the tongue apex.[1] Each of the two sensors independently tracked the movements of its associated effector, yielding a positional signal of tongue body movements relevant for the production of /ka/ and another positional signal of tongue tip movements relevant for the production of /ta/ syllables.

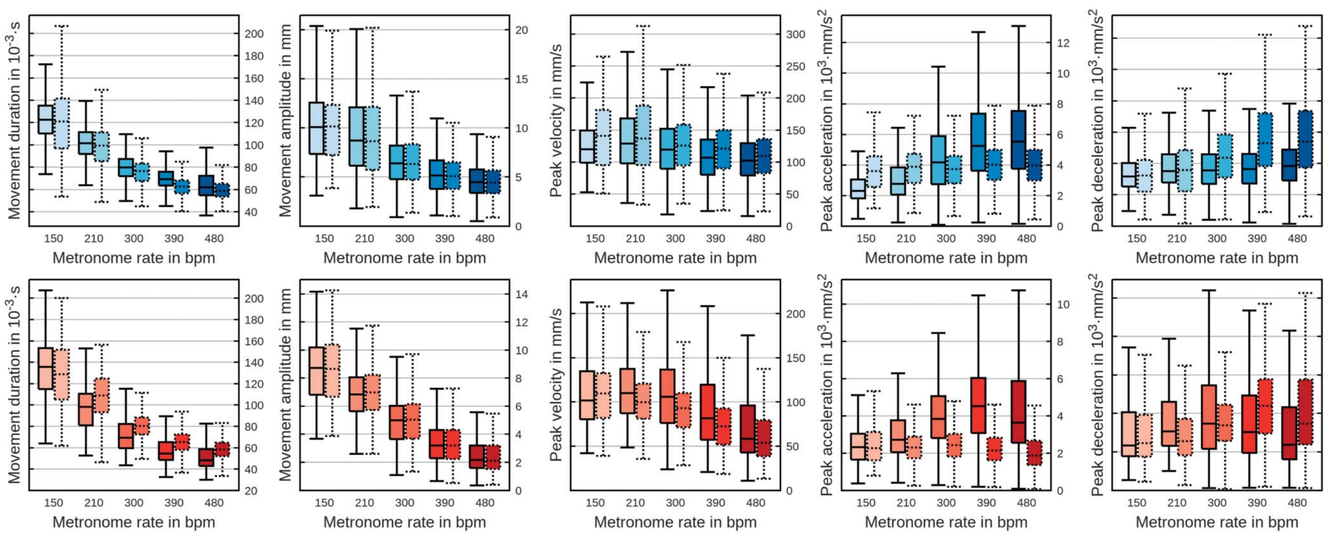
The so-registered signals were then processed twice: first, using a filtering approach with finite differences, and second, using a spline smoothing approach. In a typical application of filtering, we used a zero-phase, third-order, Butterworth, low-pass filter with a cutoff frequency of 20 Hz. From the filtered positional signals, velocity and acceleration estimates were determined by a central finite difference scheme, with an intermediary five-point average filter for the second derivative. This approach corresponds to the default smoothing implemented in the analysis software mview (authored by Mark Tiede, Haskins Laboratories), a widely used software in assessing and measuring speech kinematics (another popular software for measuring speech, SMASH, by Green et al., 2013, makes use of a similar filtering approach). For the splines approach, we used a Fortran-to-Matlab port (Kuberski, 2023) of Woltring's classical spline smoothing and differentiation code (Woltring, 1986) obtaining heptic-order splines with smooth derivatives up to the sixth order. For the expected noise present in the registered signals, we used a fixed predicted mean square error of 0.5 mm, which is within the range of values given by the manufacturer of the EMA device and those reported by other works explicitly devoted to its accuracy (Bilibajkic et al., 2015; Lezcano et al., 2020; Savariaux et al., 2017; Sigona et al., 2018; Stella et al., 2013).

Following the process of raw data smoothing by the two approaches, the resulting signals of the tongue body (for /ka/) and tongue tip (for /ta/) effectors were separately divided into movement cycles by means of their local tangential velocity minima within the full three dimensions: adjacent landmarks of minimal tangential velocity in a signal defined the principal direction of movements (cf. the body-spatial reaching axis in Saltzman & Kelso, 1987; and the constrictional dimension in Saltzman & Munhall, 1989) and spatial projections of the signal onto the direction vectors yielded one-dimensional trajectories of movements as typically employed in studies of speech kinematics (Adams et al., 1993; Munhall et al., 1985; Ostry et al., 1983, 1987). The resulting trajectories were then segmented into individual movements by a 20% peak-velocity criterion. Overall, the segmentation process yielded 5,144 closing and 5,276 opening movements for /ka/ syllables, and 4,632 closing and 4,723 opening movements for /ta/ syllables.

In a final step, we determined the set of classic kinematic parameters for each movement: movement duration (T) as the temporal difference between onset and offset, movement amplitude (A) as the positional difference between onset and offset, peak velocity (v) as the peak value of the position's first derivative (velocity) within a movement, and two values of peak acceleration (a) corresponding to the two peaks in the position's second derivative (a first peak in the acceleration phase of a movement and a second in its deceleration phase). For ease of reference, we adopted a sign convention for the values of the kinematic parameters. Whereas amplitudes and peak velocities of closing movements (motion toward a constriction) were assigned positive values, the same parameters for opening movements (motion away from a constriction)

[1]In EMA research, application of the sensors takes substantial time during the session with the participant. In our lab, approximately 30 min are needed to glue the whole set of sensors to their positions. Gluing starts with the reference sensors (nose bridge, left and right mastoids), continues with the tongue sensors (tongue dorsum, tongue tip), then the upper and lower incisor sensors, and finishes with the application of two sensors on the lips (upper and lower lips). During the entire time of application, the experimenter is in continuous conversation with the participant, who, in the process, becomes accustomed to speaking with the sensors attached.

Figure 1. Box plots showing the values of the classic kinematic parameters in the filtering approach. From left to right: movement duration, movement amplitude, peak velocity, peak acceleration, and peak deceleration. The top row shows data /ka/, and the bottom row shows data of /ta/ syllables. Within each panel, solid boxes correspond to data from closing movements and dashed boxes correspond to data from opening movements.



were assigned negative values. Similarly, peak acceleration (peak deceleration) values of closing movements were assigned positive (negative) values and peak acceleration (peak deceleration) values of opening movements were assigned negative (positive) values.

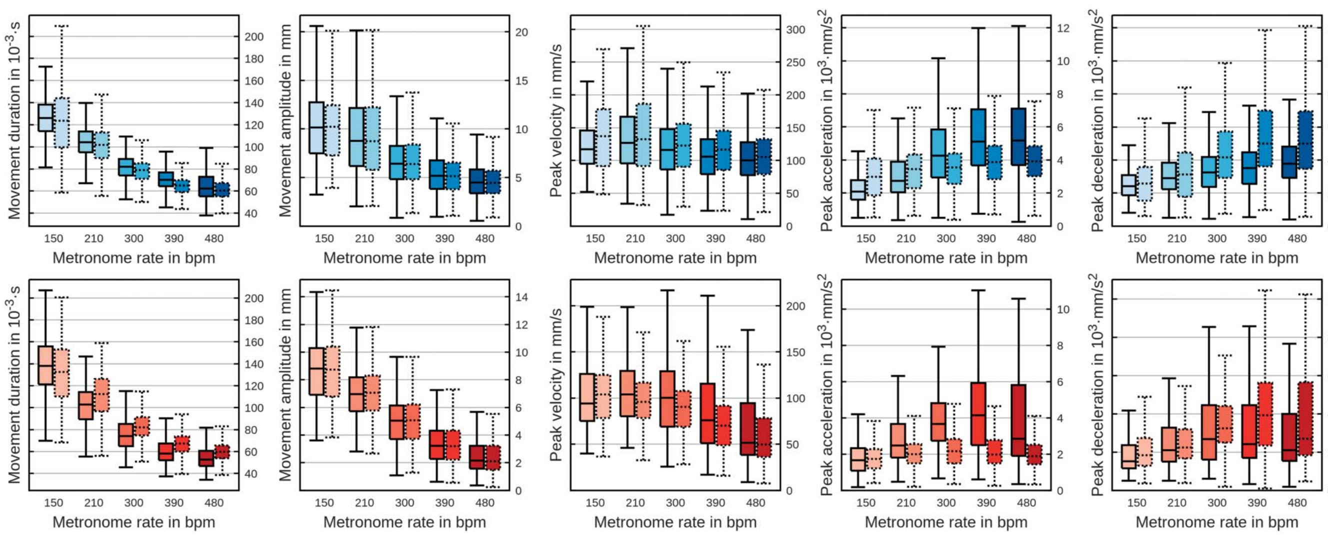
In quantifying differences in the kinematic parameters estimated from the two smoothing approaches, we used a relative percentage difference (RPD) indicator. For two estimates, x_1 and x_2, of the same quantity, RPD is given by $2\times(x_1 - x_2)/(x_1 + x_2)$ multiplied by 100%. The RPD value thus indicates by how many percent the two measures, x_1 and x_2, differ with respect to their mutual mean.[2] Of course, the mere demonstration of any differences between the two smoothing approaches does not settle the issue of which approach gives estimates closer to the kinematic reality (i.e., the true underlying values of the kinematic parameters). In fact, this kinematic reality is a priori unknown. Crucially, however, even though the true underlying values of the kinematic parameters may be unknown, what is known is that these parameters enter into certain relations with one another, referred to in the field of speech production as kinematic relations. It is to these relations that we turned to in evaluating which smoothing approach does better in reconstructing the kinematic reality. Specifically, for any given relation among the kinematic parameters, we fitted a regression model to the approach-specific parameter values. The performance difference between the two smoothing approaches was then quantified by means of standard errors of the regressions (the root-mean-squares of the regression residuals) and the related RPD values derived from the regression errors. Smaller regression errors signify a better fit of the regression model, and positive (negative) RPD values express the amount of regression performance by which one of the approaches outperforms (underperforms) the other.

Results

In a first step of our comparison of the two smoothing approaches, we assessed the amount of variation found in the kinematic parameters estimated by these approaches. Figures 1 and 2 visualize the kinematic estimates of movement duration, movement amplitude, peak velocity, and peak acceleration/deceleration (from left to right). Figure 1 shows the estimates from the filtering approach, and Figure 2 shows the estimates from the splines approach. Within the two figures, blue colors represent data of /ka/ syllables and red colors represent data of /ta/ syllables. Overall, from visual inspection of both figures, it becomes clear that our paradigm elicited movements of high kinematic variation regardless of the smoothing approach used: across all metronome rates, the kinematic parameter values cover ranges of about one order of magnitude. That is, minimal and maximal values within the parameter ranges differ by a factor of about 10. For comparison, in a study devoted to exploring the kinematics of various

[2]To give an example, consider two duration measures, T_1 and T_2, estimated for the same movement, one deriving from the filtering and the other from the spline smoothing approach. Then, the RPD of, for example, T_1 = 95 ms and T_2 = 105 ms is −10%, indicating that both estimates differ in value by 10% of their mutual mean; the negative sign of the RPD value indicates that T_1 is smaller than T_2.

Figure 2. Box plots showing the values of the classic kinematic parameters in the splines approach. From left to right: movement duration, movement amplitude, peak velocity, peak acceleration, and peak deceleration. The top row shows data /ka/ and the bottom row shows data of /ta/ syllables. Within each panel, solid boxes correspond to data from closing movements and dashed boxes correspond to data from opening movements.



different speaking conditions, Perkell et al. (2002) reported kinematic parameters of tongue movements with a range of about 0.2–0.4 orders of magnitude, corresponding to a factor of only about two.

Impressionistically, when visually comparing Figures 1 and 2, the ranges of the kinematic parameter estimates do not appear to be affected by the choice of the smoothing approach. However, when quantitatively comparing individual parameter values on a per-movement basis (i.e., comparing the approach-specific estimates for the same movements), appreciable differences between the filtering and the splines approach become apparent. Tables 1 and 2 show grand means of the RPDs between the kinematic estimates of the two approaches, separated by the direction of movement and the metronome rate. Table 1 shows data from /ka/ syllables and Table 2 shows data from /ta/ syllables. In the two tables, negative (positive) RPD values signify that the related kinematic estimates from the filtering approach are smaller (larger) than the related estimates from the splines approach. Substantial differences between the estimates from the two approaches can be observed: for both syllables /ka/ (see Table 1) and /ta/ (see Table 2), and across all speech rates (columns in the two tables), magnitudes of the RPD values are in a range of several percent or higher. In particular, averaged across all metronome rates and the two movement directions, differences in the kinematic parameter estimates attain values of −2.4% (*SD* = 1%) for movement duration, −0.7% (*SD* = 0.4%) for movement amplitude, 3.4% (*SD* = 1.7%) for peak velocity, 6.5% (*SD* = 5.2%) for peak acceleration, and 12.0% (*SD* = 8.3%) for peak deceleration.

As demonstrated in terms of the RPD values in Tables 1 and 2, the different signal reconstruction approaches result in substantially different estimates of the (same) kinematic parameters.[3] However, as anticipated in the Method section, this result does not settle the issue of which approach does better in reconstructing the speech movements of our participants. To address this aim, we turned to the fact that the kinematic parameters of speech show coherent relations to one another. Specifically, three such relations are well-documented in the speech production literature. The first relation states that peak velocity of a movement is proportional to its average velocity ($v \sim A/T$; Munhall, 1984; Munhall et al., 1985; Ostry & Munhall, 1985; Perkell et al., 2002). The second relation expresses another proportionality between a movement's peak velocity and its amplitude ($v \sim A$), with a slope dependent on speech rate (Kelso et al., 1985; Kühnert & Hoole, 2004; Ostry & Munhall, 1985; Vatikiotis-Bateson & Kelso, 1993). The third relation states that peak acceleration of a movement is

[3]The significance of an RPD value depends on the specific context of the comparison. There is no context-free or domain-general RPD value beyond which a difference is said to be significant (as is the case also for other relative measures). RPD values are significant when the difference they represent is somehow meaningful in the context where that difference is judged. In our case, the estimation of kinematic parameters from articulatory signals (i.e., movement duration, movement amplitude, peak velocity values), differences of several percent between the two smoothing approaches, are certainly meaningful in the context of works that require for their examination, the establishment of differences in the kinematic parameter values (e.g., vowel height can show differences of less than 1 mm in movements of 10 mm amplitude; cf. Cunha & Hoole, 2017; Lee et al., 2015; Ratko et al., 2023).

Table 1. Differences in the kinematic parameter estimates of /ka/ syllables (relative percentage difference, in %).

/ka/ syllables		Metronome rate in bpm				
		150	210	300	390	480
Duration *T*	Closing	–2.9	–2.4	–1.9	–0.5	–0.4
	Opening	–2.3	–2.4	–2.4	–2.6	–1.9
Amplitude *A*	Closing	–1.0	–0.9	–1.0	–0.4	–0.3
	Opening	–0.6	–0.7	–0.9	–0.6	–0.4
Peak velocity *v*	Closing	2.2	2.3	2.5	0.9	0.9
	Opening	2.5	2.1	2.8	3.0	3.3
Peak acceleration *a*	Closing	2.7	1.4	0.8	2.6	4.0
	Opening	11.3	8.2	5.0	3.9	1.4
Peak deceleration *a*	Closing	23.7	17.1	9.3	4.1	1.2
	Opening	9.1	6.3	3.4	4.2	5.5

Note. Negative (positive) values signify that the estimates of the filtering approach are smaller (larger) than the estimates of the splines approach.

proportional to the movement's amplitude ($a \sim A$, closely related to Hooke's law), also with a slope dependent on speech rate (Kelso et al., 1985).

Figures 3 and 4 show scatter plots of the three kinematic relations. Figure 3 depicts these relations using parameter values from the filtering approach and Figure 4 using parameter values from the splines approach. In the two figures, the different experimental conditions are color-coded, with data from /ka/ syllables indicated by blue and data from /ta/ syllables indicated by red colors; fainter (darker) shades in the figures correspond to slower (faster) metronome rates. Due to our sign convention for the kinematic parameters introduced earlier, data from movements of different directions and data of acceleration and deceleration phases separate into different quadrants of the shown coordinate systems (as indicated by the labels in the figures' quadrants). Inspection of the two figures reveals clear systematicities, as expected from the three just-outlined kinematic relations: the left column depicts the proportionality between peak velocity v and average velocity A/T, the middle column depicts the rate-dependent proportionality between peak velocity v and movement amplitude A, and the right column shows the other rate-dependent proportionality between peak acceleration a and movement amplitude A.

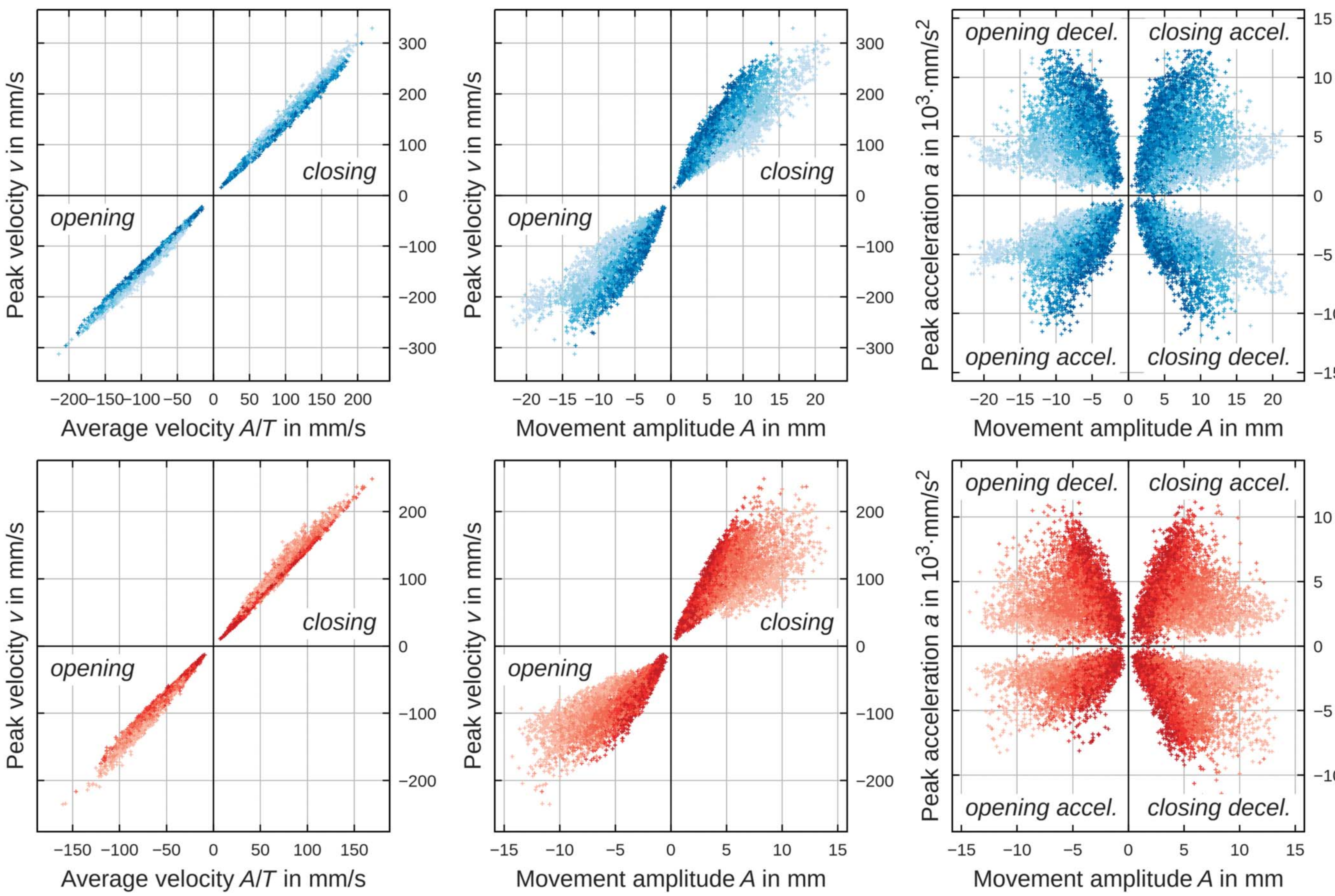
Merely visually comparing Figures 3 and 4 does not suggest a qualitative difference between the filtering and the splines approach in capturing the structure of the three relations. Yet, a proper evaluation of the two smoothing approaches requires a statistical assessment of how well the data derived from these approaches reflect the kinematic relations (which we know must exist in the data). Thus, we conducted regressions based on the functional form of the three relations ($v \sim A/T$, $v \sim A$, and $a \sim A$) with speech rate–dependent slopes (as color-coded in Figures 3 and 4) and the respectively related quantities (T, A, v, and a) reconstructed from the two smoothing approaches. The resulting correlation strengths (R^2, using gnuplot's fit functionality; Williams & Kelly, 2021) were in the range of about 0.75–0.98. All p values resided

Table 2. Differences in the kinematic parameter estimates of /ta/ syllables (relative percentage difference, in %).

/ta/ syllables		Metronome rate in bpm				
		150	210	300	390	480
Duration *T*	Closing	–2.6	–4.0	–4.4	–3.9	–3.2
	Opening	–2.7	–2.6	–2.3	–1.8	–1.6
Amplitude *A*	Closing	–0.9	–1.7	–1.7	–0.4	–0.1
	Opening	–0.9	–0.8	–0.8	–0.2	–0.1
Peak velocity *v*	Closing	5.5	6.1	6.1	5.8	6.1
	Opening	4.6	3.4	2.9	2.3	3.1
Peak acceleration *a*	Closing	11.6	6.3	4.2	7.0	14.7
	Opening	18.8	13.6	8.3	2.4	0.8
Peak deceleration *a*	Closing	28.9	26.2	21.4	14.6	6.7
	Opening	19.4	9.7	5.8	8.9	14.8

Note. Negative (positive) values signify that the estimates of the filtering approach are smaller (larger) than the estimates of the splines approach.

Figure 3. Scatter plots showing the three kinematic relations in the filtering approach. From left to right: $v \sim A/T$, $v \sim A$, and $a \sim A$. The top row shows data of /ka/ and the bottom row shows data of /ta/ syllables; speech rate is color coded, with fainter (darker) shades corresponding to movements of slower (faster) rates. decel. = deceleration; accel. = accelaration.

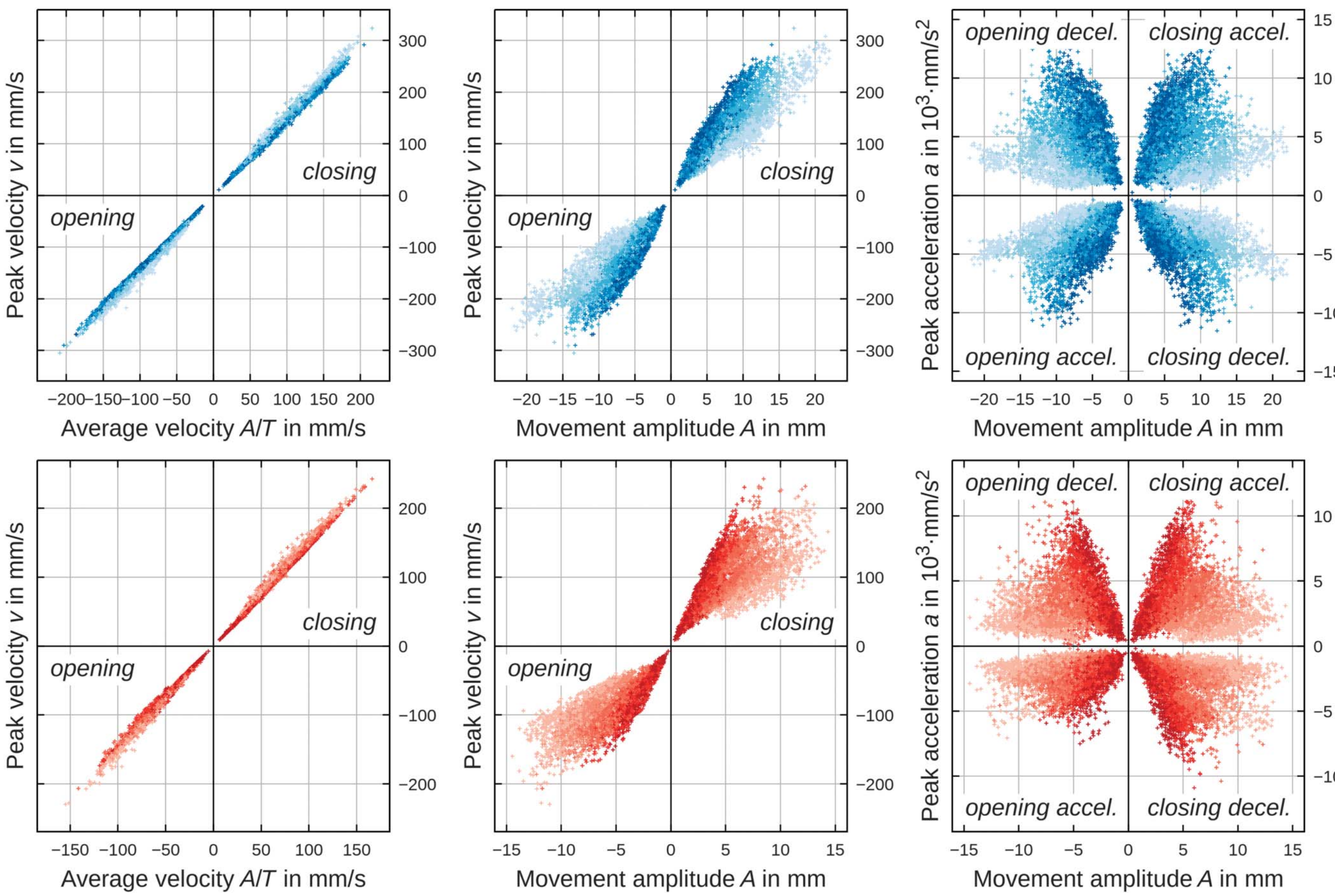


below .001, and the regressions showed solid residual statistics, both indicating strong evidence for the presence of the three kinematic relations regardless of the smoothing approach used.[4] To quantitatively compare the relative performance of the two approaches, we turn to the standard errors of the regressions (the root-mean-squares of the regression residuals), as well as the related RPD values derived from the regression errors. Smaller regression errors signify a better fit to the regression model, and negative (positive) RPD values express the amount of regression performance by which the splines approach outperforms (underperforms) the filtering approach. Table 3 shows these results, separated by the type of the kinematic relation, the syllable produced, and the direction of movement. The values shown in Table 3 make clear that for almost all regressions conducted, the regression errors of the splines are substantially smaller than those of the filtering approach, with RPD values in the range of −11.2% to −42.8% and an average of −26.2%. There is only one case (in the $a \sim A$ relation of /ta/ opening movements) in which the filtering approach shows a numerically lower error than that of the splines. However, the difference in regression errors is only about 3%, which seems negligible given the fact that in all other cases, splines outperform the filtering approach with much higher RPD magnitudes.

In a final step of our assessment of the two smoothing approaches, we considered potential differences in their performance with respect to the two languages of our participants (German and English). Specifically, we

[4]To evaluate the applicability of regression models for the three kinematic relations, we inspected distributions of the regression residuals and quantified the amount of their heteroscedasticity by a Breusch–Pagan test (i.e., R^2 values of an auxiliary regression between the squared residuals and the regressors, with smaller values indicating less heteroscedasticity). For both smoothing approaches (filtering and splines), normality of the regression residuals was visually attested. For the filtering approach, the Breusch–Pagan test returned auxiliary R^2 values in the range of .012–.233 with an average of .097. The same test carried out for the splines approach yielded values in the range of .003–.304 with an average of .101. All p values resided below .001. Thus, for both smoothing approaches, the regression models of the three kinematic relations met the requirements for statistical evaluation.

Figure 4. Scatter plots showing the three kinematic relations in the spline smoothing approach. From left to right: $v \sim A/T$, $v \sim A$, and $a \sim A$. The top row shows data of /ka/ and the bottom row shows data of /ta/ syllables; speech rate is color coded, with fainter (darker) shades corresponding to movements of slower (faster) rates. decel. = deceleration; accel. = accelaration.



performed the above reported regression procedure again and evaluated the same statistical metrics as before, but this time, separately for the German and English subsets of the data. In the resulting correlation strengths of the three kinematic relations, we found no substantial differences between the two languages: R^2 values of the regressions of the German subset were in the range of .71–.98, and R^2 values of the English subset were in the range of .70–.98. For both languages, all regressions yielded p values below .001 and the related residuals showed no conflict with the regression assumptions.[5] We likewise found no substantial differences between the two languages regarding the two performance measures of standard regression error and RPD expressed in Table 4 (German subset) and Table 5 (English subset). For both languages, the regression errors of the splines are substantially smaller than those of the filtering approach. Corresponding RPD values are in the range of −9.0% to −63.8%, with an average of −25.3% for the German subset and an average of −30.2% for the English subset. Overall, the correlation strengths of the three kinematic relations show no dependence on language and the splines approach overwhelmingly outperforms the filtering approach in terms of regression errors.

[5]Normality of the regression residuals was visually attested, for both languages (German and English) and both smoothing approaches (filtering and splines). For German, the Breusch–Pagan test of heteroscedasticity returned auxiliary R^2 values in the range of .021–.216 (filtering) and .010–.284 (splines), with averages of .085 (filtering) and .113 (splines). For English, the same test returned values in the range of .012–.270 (filtering) and .017–.322 (splines), with averages of .108 (filtering) and .110 (splines). Thus, overall, the regression models of the three kinematic relations met the requirements for statistical evaluation.

Discussion

In nonspeech areas of human movement science, reconstructing movement signals from measurement devices has employed, for nearly four decades now, a splines approach, with the speech community lagging somewhat

Table 3. Full data set: standard errors of the regressions for the three kinematic relations (v ~ A/T, v ~ A, and a ~ A) and the related relative percentage difference (RPD) values between the splines and the filtering approach.

Full data set		v ~ A/T		v ~ A		a ~ A (acceleration phase)		a ~ A deceleration phase)	
		Error	RPD in %	Error in 1/s	RPD in %	Error in 1/s^2	RPD in %	Error in 1/s^2	RPD in %
/ka/ closing	Filter	0.074	−20.9	0.838	−26.5	260.4	−33.2	140.9	−11.2
	Spline	0.060		0.642		186.3		125.9	
/ka/ opening	Filter	0.069	−17.3	0.707	−23.2	123.8	−13.2	164.3	−23.8
	Spline	0.058		0.560		108.5		129.4	
/ta/ closing	Filter	0.097	−39.5	1.154	−44.9	318.1	−42.8	161.0	−34.5
	Spline	0.065		0.731		206.0		113.6	
/ta/ opening	Filter	0.084	−35.0	0.972	−32.7	183.0	3.1	345.7	−24.5
	Spline	0.059		0.699		188.8		270.2	

Note. Smaller errors signify a better fit to the regression model, and negative (positive) RPD values express the amount of regression performance by which the splines approach outperforms (underperforms) the filtering approach.

behind in keeping with the more antiquated digital filtering and finite differences approach. In an aim to inform the speech community about the existence and performance of the splines approach, we compared kinematic parameter values extracted from the two approaches using data of speech movements from /ka/ and /ta/ syllables spoken under different rates by speakers of German and English.

In comparing the two smoothing approaches, we first evaluated differences in their estimates of a classic set of kinematic parameters. Although the ranges of the kinematic parameter estimates are not affected by the choice of the smoothing approach (see Figures 1 and 2), juxtaposing individual kinematic parameters on a per-movement basis revealed substantial differences in the estimated values of the two approaches. Quantified using the RPD metric, which is appropriate in our case given the multiplicity of kinematic parameters and their expression in different physical dimensions, estimated parameter differences are in the range of several percent and higher (see Tables 2 and 3). A notable characteristic of this RPD comparison is that higher order derivatives are estimated with higher differences between the two approaches. To wit, movement amplitude (zeroth-order derivative) values attain RPD magnitudes of about 1%, peak velocity (first-order derivative) values attain RPDs of about 3%, and peak acceleration and peak deceleration (second-order derivative) values attain even higher RPDs of about 9%. This pattern in the results is not unexpected. Derivatives of noisy signals are highly sensitive to the signal's noise characteristics (the higher the order of differentiation, the more so), and these characteristics behave very different in the two smoothing approaches: whereas the noise characteristics in the filtering approach progressively change with each order of differentiation (because of the repeated iteration of filtering and finite differences, each effectively acting as a low-pass/high-

Table 4. German subset: standard errors of the regressions for the three kinematic relations (v ~ A/T, v ~ A, and a ~ A) and the related relative percentage difference (RPD) values between the splines and the filtering approach.

German subset		v ~ A/T		v ~ A		a ~ A (acceleration phase)		a ~ A (deceleration phase)	
		Error	RPD in %	Error in 1/s	RPD in %	Error in 1/s^2	RPD in %	Error in 1/s^2	RPD in %
/ka/ closing	Filter	0.075	−17.4	0.822	−25.2	230.4	−39.7	132.2	−14.1
	Spline	0.063		0.638		154.1		114.8	
/ka/ opening	Filter	0.081	−9.0	0.756	−14.9	122.0	−19.4	120.8	−25.9
	Spline	0.074		0.651		100.4		93.1	
/ta/ closing	Filter	0.098	−54.5	1.180	−59.9	276.1	−63.8	135.0	−22.8
	Spline	0.056		0.636		142.5		107.4	
/ta/ opening	Filter	0.087	−43.4	0.961	−45.5	152.9	21.0	310.0	−48.6
	Spline	0.056		0.605		188.8		188.8	

Note. Smaller errors signify a better fit to the regression model, and negative (positive) RPD values express the amount of regression performance by which the splines approach outperforms (underperforms) the filtering approach.

Table 5. English subset: standard errors of the regressions for the three kinematic relations ($v \sim A/T$, $v \sim A$, and $a \sim A$) and the related relative percentage difference (RPD) values between the splines and the filtering approach.

English subset		$v \sim A/T$		$v \sim A$		$a \sim A$ (acceleration phase)		$a \sim A$ (deceleration phase)	
		Error	RPD in %	Error in 1/s	RPD in %	Error in $1/s^2$	RPD in %	Error in $/1/s^2$	RPD in %
/ka/ closing	Filter	0.074	−24.2	0.850	−29.6	279.3	−30.7	138.6	−13.6
	Spline	0.058		0.631		205.0		120.9	
/ka/ opening	Filter	0.055	−29.2	0.665	−33.3	122.6	−12.6	178.8	−24.4
	Spline	0.041		0.475		108.1		139.9	
/ta/ closing	Filter	0.096	−27.2	1.119	−31.4	353.1	−33.5	182.3	−41.5
	Spline	0.073		0.815		251.9		119.7	
/ta/ opening	Filter	0.081	−26.6	0.982	−23.7	205.1	−10.4	372.3	−12.9
	Spline	0.062		0.774		184.9		327.2	

Note. Smaller errors signify a better fit to the regression model, and negative (positive) RPD values express the amount of regression performance by which the splines approach outperforms (underperforms) the filtering approach.

pass filter modifying the noise left in the signal for the next step), the noise characteristics in the spline smoothing approach are held constant across the different orders of differentiation (splines handle the two processes of smoothing and differentiation at once, using only a single fixed root-mean-square error in characterizing the noise). The demonstration of such differences is an important result in itself; various hypotheses in speech science require for their examination, the establishment of differences in kinematic parameter values, and, as demonstrated in terms of the RPD metric here, the different signal reconstruction approaches result in appreciably different parameter values.

Another important aspect of our study is that, beyond having established substantial differences between the two smoothing approaches, we also addressed the issue of which approach provides better kinematic parameter estimates. Here, we harnessed the fact that the kinematic parameters in speech and any other skilled action are not independent from one another (Sejnowski, 1998). Specifically, laws or mutual relations hold between the parameters such that if one parameter changes in some way, the rest of the parameters must adjust to that change in attestation of the reciprocity dictated by the existence of their relation. In harnessing the presence of these laws for our purposes, at issue is not the extent of the differences in isolated parameter values across the two approaches, but rather how tightly related, within each approach, the parameter values are as per the functional form of the law dictating their relation. Take, for example, the $v \sim A/T$ relation. The more tightly the cloud of estimated parameter values (for the three parameters of peak velocity, movement amplitude, and movement duration that enter into this relation) hugs the linear relationship expressed by this law, the better the signal reconstruction approach. It is this degree of conformity to mutual relations among the parameters that we have judged in our results using regression errors and their RPD values. Any deficiencies of a smoothing approach should erode the quality of the regression by introducing irregular variation in the isolated parameters that is not caused by the correlation represented by the kinematic relation at hand. Overall, our results indicate that the splines approach clearly outperforms the filtering approach in that its parameter estimates consistently conform better to the correlations expressed by the three kinematic relations under evaluation. This is evidenced by the smaller regression errors presented in Tables 3–5, with average RPD values in the range of −25.3% to −30.2%. There are only two out of 48 cases across the three tables where the filtering approach shows a numerically lower regression error than that of the splines. Given the small proportion they represent within the set of all regressions performed, we regard these two isolated cases as outliers.

We turn next to highlight a distinction between this work and prior assessments of signal reconstruction approaches in speech. So far, to our knowledge, other work on the reconstruction of articulatory kinematics (e.g., Berry, 2011; Bilibajkic et al., 2015; Kröger et al., 2008; Kroos, 2012; Lezcano et al., 2020; Savariaux et al., 2017; Sigona et al., 2018; Stella et al., 2013) has focused primarily on issues of accuracy of the devices used to register articulatory data (e.g., the Carstens AG device series, and the NDI Wave and Aurora systems). For that reason, these works have used artificially generated movements (e.g., disc rotations) and evaluated how accurately the predetermined characteristics of these movements show through in reconstructed signals from the different devices. Our contribution departs from this line of work. Instead of assessing device accuracy, our work compares smoothing approaches for speech signals after they have been registered with any device. Though it is certainly possible, also for our purposes, to make use of artificially generated movements and our choice of not doing so may be seen as a limitation of this work, we note that the kinematic laws that govern simple mechanical systems (e.g., rotating discs) are of a different nature from those that govern movements of complex biological systems (e.g., human tongues). We are not aware of any laws or theory in the literature that can be used to relate the performance of different signal reconstruction approaches across artificially generated and natural speech movements, and ultimately, the most important scenario that matters to speech science is that of movements generated by humans where the trajectories are those of moving tongues as opposed to, for example, moving discs. Thus, in comparing different approaches to signal reconstruction, we opted for using natural speech and specifically the structure of empirically well-documented kinematic relations in speech instead of a structure induced by artificial movements.

Finally, in anticipation of reservations with using splines in day-to-day laboratory work, we close by addressing potential limitations of splines. One potential drawback of using the splines approach is computational expense. Longer data processing run times are to be expected when using this method. Observations with our data sets indicate that processing signals with the splines approach may extend computational time by a factor of three to five compared to the filtering approach. However, whereas longer processing might have been an issue in the 1980s, during a time when the splines approach gained traction in biological signal processing, nowadays, this is hardly an issue: present-day laptop computers easily outperform supercomputers from that time in terms of relevant measures such as memory access, power consumption, floating point performance, and so forth. Furthermore, in analysis pipelines, smoothing is typically applied only once, followed by an extended period of data labeling and quantification whose time dwarfs the time for processing signals with the splines approach. Overall, we see no limitations in switching from a filtering to a splines approach, except perhaps for concomitant issues associated with changing code in existing data processing pipelines.

Conclusions

Using speech movement data registered with EMA, we compared two approaches of reconstructing articulatory signals, a digital filtering with approximation of derivatives by finite differences approach and a spline smoothing approach. In particular, we compared values of the kinematic parameters of movement duration, movement amplitude, peak velocity, peak acceleration, and peak deceleration, as estimated from these approaches. We furthermore assessed, within each approach separately, how well the approach-specific parameter values conform to empirically known laws about how these parameters should be related to one another. The kinematic parameter estimates show significant differences between the two approaches, with RPDs in the range of several percent and higher. In the regression performance of the two approaches, we observed substantially smaller regression errors for the splines than for the filtering approach, as evidenced by RPD values in the range of −25% to −30%. This result is in broad agreement with several reports from other fields of movement science and underpins the superiority of the splines approach also in the domain of speech.

Data Availability Statement

The data that support the findings of this work are available from the corresponding author, Stephan R. Kuberski, upon reasonable request.

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