DYNAMICS AND KINEMATICS OF REPETITIVE SPEECH MOVEMENTS

Adamantios I. Gafos, Stephan R. Kuberski

University of Potsdam
Linguistics Department
Excellence Area in Cognitive Science
Potsdam, Germany

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Linear model kinematics (1)

- using the **standard model** of speech gestures (Saltzman and Munhall, 1989) we can model a (one-dimensional) **single articulator movement** by the linear second order system

$$\ddot{x} = -kx - b\dot{x}\,, \quad b = 2\zeta\sqrt{k}$$

- model parameters **stiffness** k and **damping** b can be defined in terms of a dimensionless damping ratio $\zeta \geq 0$ and natural frequency $\omega_0 = \sqrt{k}$
- we can solve the differential equation yielding expressions for **displacement**, **velocity** and **acceleration** (dependent on damping ratio)
- e.g., for **critical damping** ($\zeta = 1$) with initial conditions $(x_0, \dot{x}_0)$

$$\begin{aligned} & x(t) = \mathrm{e}^{-\omega_0 t}\,(x_0 + [\omega_0 x_0 + \dot{x}_0]t) \\ \Rightarrow \quad & \dot{x}(t) = \mathrm{e}^{-\omega_0 t}\,(\dot{x}_0 - \omega_0[\omega_0 x_0 + \dot{x}_0]t) \\ \Rightarrow \quad & \ddot{x}(t) = \mathrm{e}^{-\omega_0 t}\left(-2\omega_0\dot{x}_0 - \omega_0^2 x_0 + \omega_0^2[\omega_0 x_0 + \dot{x}_0]t\right) \end{aligned}$$

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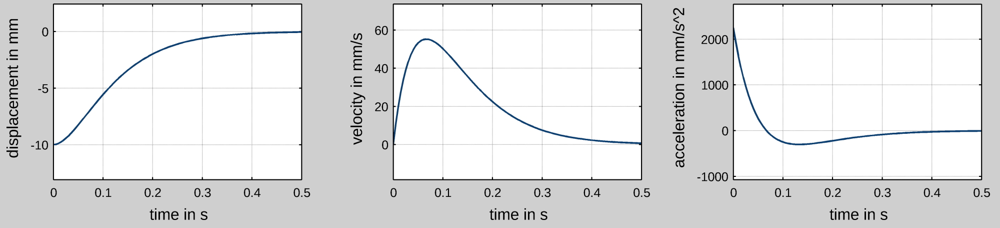
Linear model kinematics (2)

- example curves of **displacement** $x(t)$, **velocity** $\dot{x}(t)$ and **acceleration** $\ddot{x}(t)$ for the **critically damped** linear system ($x_0 = -10\,\mathrm{mm}$, $\dot{x}_0 = 0\,\mathrm{mm/s}$, $k = 225\,\mathrm{s}^{-2}$)



- a simple model of an articulatory movement defines the **kinematic variables**:

1. **peak velocity** v^* at instant of zero acceleration
2. movement onset and offset at instants of a certain fraction of peak velocity
3. movement **amplitude** A and **duration** T follow straight forward
4. as well as **relative time to peak velocity** (RTTP)

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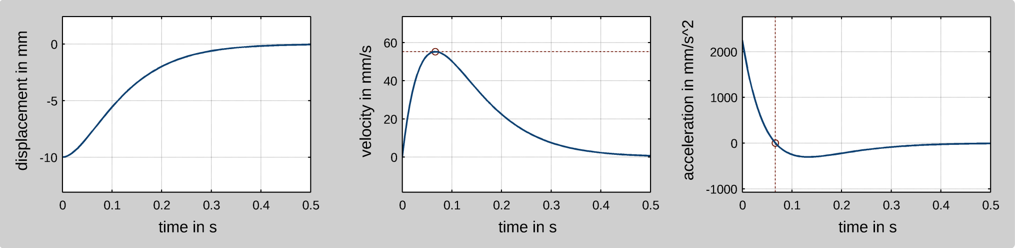

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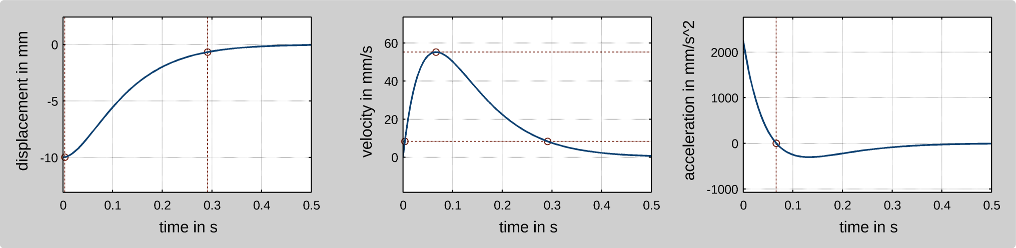

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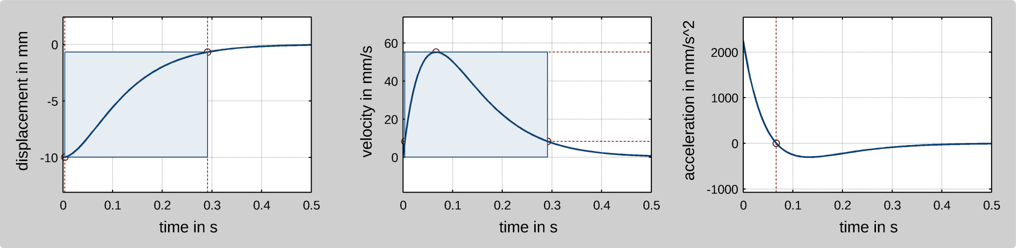

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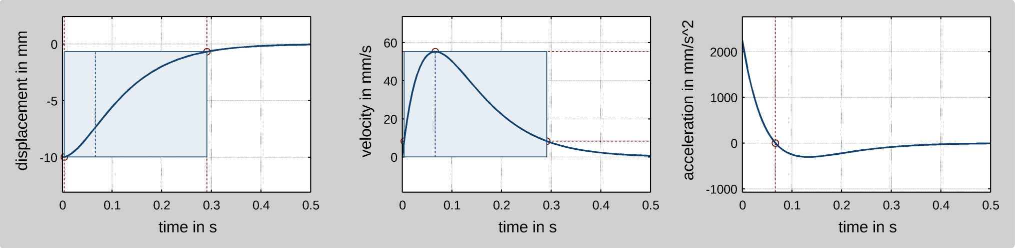

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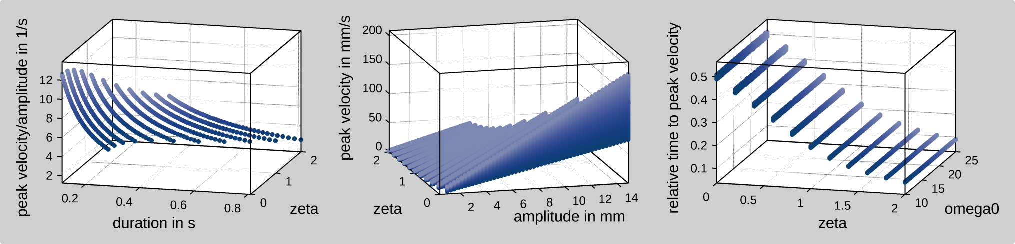
Linear model kinematics (3)

- analytical treatment of the **standard model** leads to the **kinematic relations**:

$$\frac{v^*}{A} = \frac{c\pi}{T} \quad \text{and} \quad v^* = c\omega_0 A$$

- **constant of proportionality** c depends on damping ratio ζ and is maximal ($c = 1/2$) in undamped case (simple harmonic oscillator)



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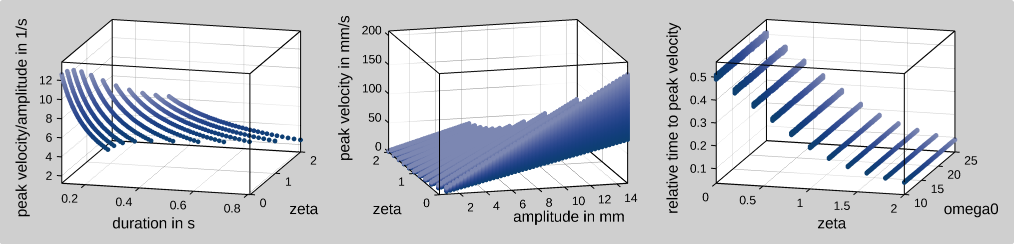

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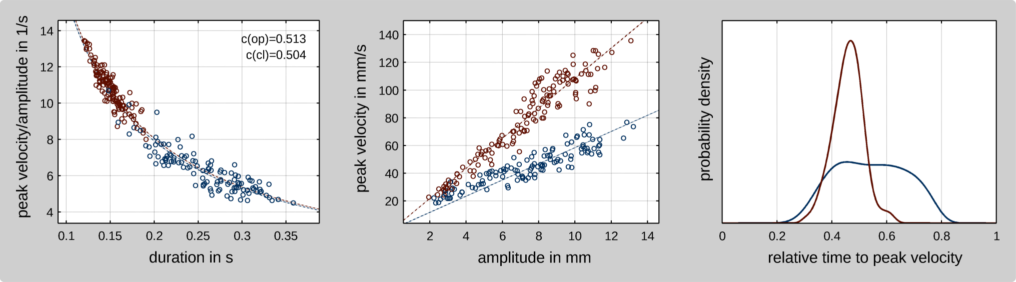
Harvard-Haskins database

- the Harvard-Haskins database of **regularly timed speech** (Patel et al., 1999) contains articulatory data of **self-paced** tongue, lips and jaw movements in /baCa.../ sequences at approx. 2 Hz
- the principal component analysis of **jaw movements** in /baba.../ (3 native English speakers, 4 trials, 11 syllables each) shows the following **kinematic characteristics**:



- aside from the asymmetry between opening (red) and closing movements (blue)

1. **kinematic relations** (left and middle) are in general agreement with the **standard model predictions**
2. high **relative time to peak velocity values** (> 0.5, right) cannot be rendered by it

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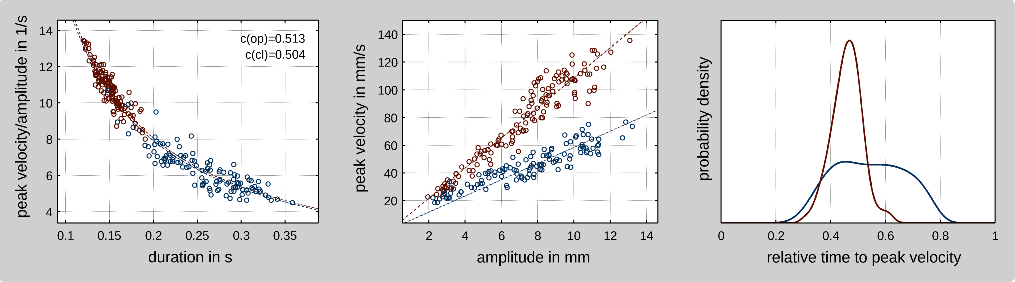

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Beyond linear fixed point dynamics

- some improvements to **fixed point dynamics** of the standard model in speech domain:
  1. explicit time dependencies/nonautonomy (e.g., gestural force field by Kröger et al., 1995)
     $$\ddot{x} = -\omega^2 x - 2\omega\dot{x}\,, \quad \text{gestural force function } \omega(t)$$
  2. additional nonlinear terms (e.g., soft spring model by Sorensen and Gafos, 2016)
     $$\ddot{x} = -kx - b\dot{x} + dx^3$$
- there exist other models from general motor control with **qualitatively different dynamics**:
  3. nonlinear oscillator by Schöner (1990) ($\omega = \omega_0 +$ "behavioral information")
     $$\ddot{x} = -(a^2 + \omega^2)x + 2a\dot{x} - 4bx^2\dot{x} + 2abx^3 - b^2x^5$$
  4. excitable system by Jirsa and Kelso (2005) with timing constant τ and external input I
     $$\begin{cases} \dot{x} = (x + y - \frac{1}{3}x^3)\tau \\ \dot{y} = -(x - a + by - I)/\tau \end{cases}$$
- 1 and 2 here can be treated equivalently in terms of kinematic relations
- 3 and 4 have not been applied to speech yet

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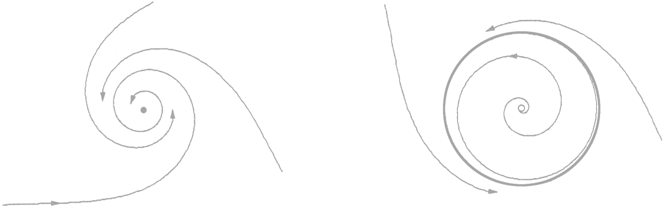
Stability and bifurcation

- all these systems are **second order systems**; a **two-dimensional phase space** is conjectured to be sufficient to render both discrete and cyclic movements (Vatikiotis-Bateson and Kelso, 1990)
- some of these models admit limit cycle dynamics (which have not been applied to speech yet)
- **fixed point** and **limit cycle attractors** differ with respect to their topology (Strogatz, 2014):



- fixed point dynamics characterizes **targeted, discrete movements**, whereas limit cycle dynamics characterizes **stationary, periodic movements**
- systems containing multiple attractors show the phenomenon of **bifurcation**, that is a **qualitative change in model behaviour** on change of some model parameter
- there are hints that (speech) rate is such a bifurcation parameter (Tuller and Kelso, 1991; Goldstein et al., 2007; Huys et al., 2008)

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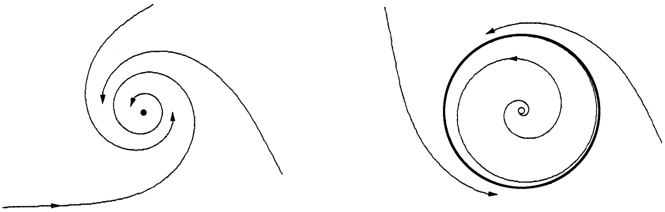

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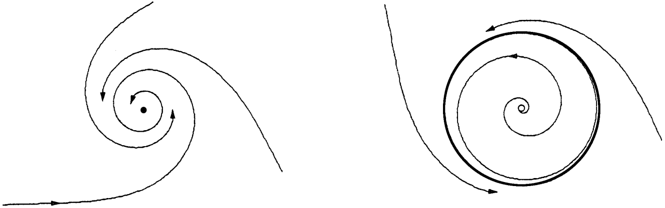

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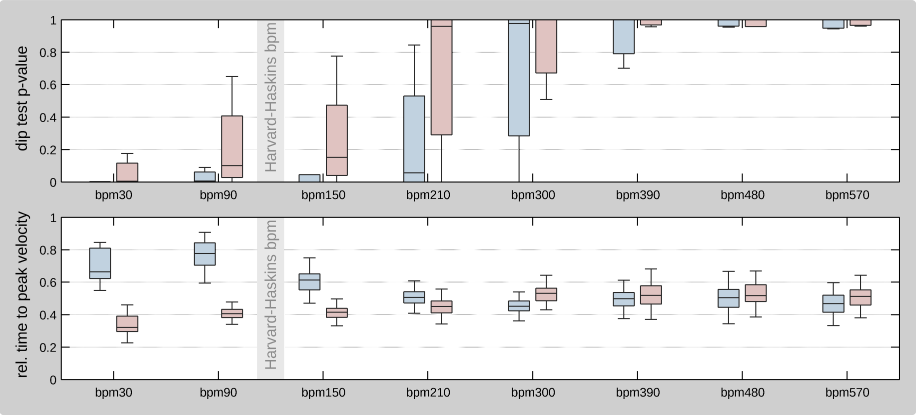
Potsdam KORSA pilot

- **repetitive speech** paradigm using systematically **controlled speech rate** (by metronome)
- pilot EMA data of tongue and jaw movements in **repetitions of CV** and CVC sequences
- native English speaker, 4 trials, each 15-30 syllables; here: **tongue tip** in /tata.../:



- properties show a **qualitative change** at 210 bpm:

multimodal	⟶	unimodal velocity profiles
two classes	⟶	single class of movements

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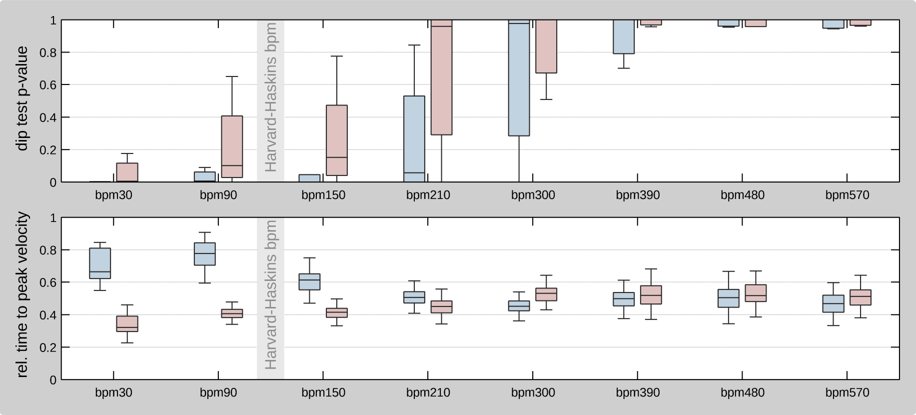

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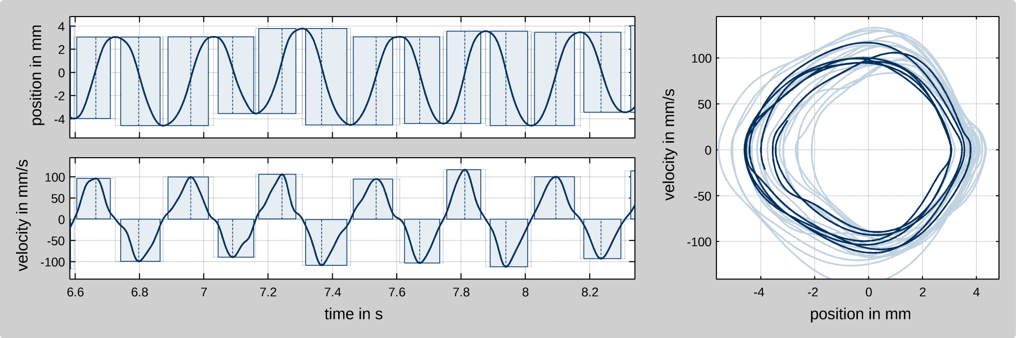
Conclusion: evidence for limit cycle

- example of tongue tip trajectory at the critical rate of 210 bpm



- evidence for two distinct timing mechanisms:
  1. dynamics at **low rates** (below critical) conform to **fixed point dynamics**
  2. dynamics at **high rates** (above critical) are **cyclic** and not governed by fixed points
- the **qualitative change** can be identified as bifurcation with **speech rate** as bifurcation parameter
- further investigations have to verify the topological structure (limit cycle)

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