Degrees of "in", "out" and "undecided" in Argumentation Networks

D. M. Gabbay and O. Rodrigues

The 6th International Conference on
Computational Models of Argument
14 September 2016

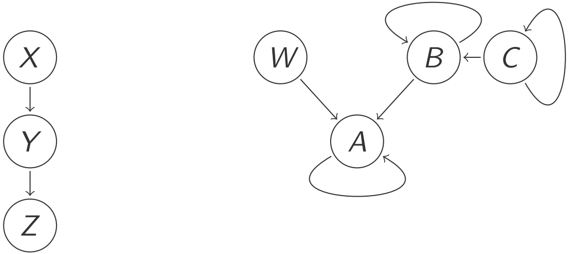
Department of Informatics
King's College London and
Bar Ilan University, Israel

Outline of the talk

1. Motivation
2. Background
3. Results
4. Conclusions and Future Work

Consider the Following Argumentation Network

$\langle \mathbf{S}, \mathbf{R} \rangle$:



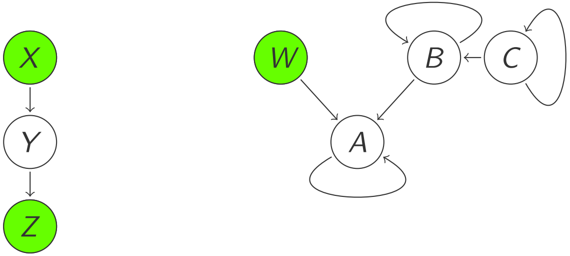
$\langle S, R \rangle$ has a unique extension $\mathcal{E} = \{X, W, Z\}$, with

$$X, W, Z = \mathbf{in}, \quad Y, A = \mathbf{out}, \quad \text{and} \quad B, C = \mathbf{und}.$$

We would like to distinguish the acceptance status of these nodes

Consider the Following Argumentation Network

$\langle \mathbf{S}, \mathbf{R} \rangle$:



$\langle S, R \rangle$ has a unique extension $\mathcal{E} = \{X, W, Z\}$, with

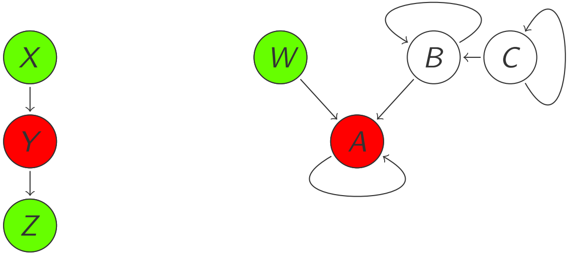
$$X, W, Z = \textbf{in}, \quad Y, A = \textbf{out}, \quad \text{and} \quad B, C = \textbf{und}.$$

We would like to distinguish the acceptance status of these nodes

- the nodes that are **in**: X, W from Z;

Consider the Following Argumentation Network

$\langle \mathbf{S}, \mathbf{R} \rangle$:



$\langle S, R \rangle$ has a unique extension $\mathcal{E} = \{X, W, Z\}$, with

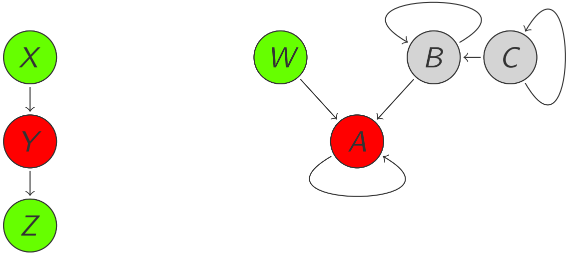
$$X, W, Z = \mathbf{in}, \quad Y, A = \mathbf{out}, \quad \text{and} \quad B, C = \mathbf{und}.$$

We would like to distinguish the acceptance status of these nodes

- the nodes that are **in**: X, W from Z;
- the nodes that are **out**: Y and A;

Consider the Following Argumentation Network

$\langle \mathbf{S}, \mathbf{R} \rangle$:



$\langle S, R \rangle$ has a unique extension $\mathcal{E} = \{X, W, Z\}$, with

$$X, W, Z = \mathbf{in}, \quad Y, A = \mathbf{out}, \quad \text{and} \quad B, C = \mathbf{und}.$$

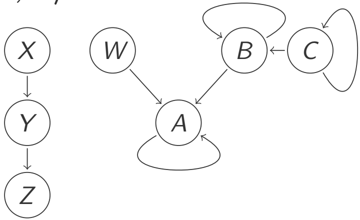
We would like to distinguish the acceptance status of these nodes

- the nodes that are **in**: X, W from Z;
- the nodes that are **out**: Y and A;
- the nodes that are **und**: B and C.

Gabbay's Equational Approach to Argumentation

Nodes get value in $[0, 1]$ and we write equations for the nodes in the network according to a particular schema tailored to the application at hand:

$\langle \mathbf{S}, \mathbf{R} \rangle$:



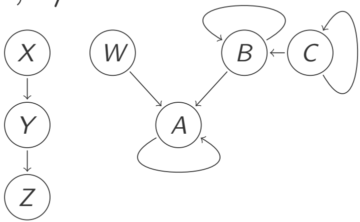
Let $Att(N) = \{Y_1, \ldots, Y_k\}$. The following two scheme define the value of a node N as:

$$(\mathbf{Eq_{max}}) \quad N = 1 - \max\{Y_1, \ldots, Y_k\}$$
$$(\mathbf{Eq_{inv}}) \quad N = \prod_{i=1}^{k}(1 - Y_i)$$

Gabbay's Equational Approach to Argumentation

Nodes get value in $[0, 1]$ and we write equations for the nodes in the network according to a particular schema tailored to the application at hand:

$\langle \mathbf{S}, \mathbf{R} \rangle$:



Example for node A:

Eq_{max}: $A = 1 - \max\{W, A, B\}$

Eq_{inv}: $A = (1-W)(1-A)(1-B)$

Let $Att(N) = \{Y_1, \ldots, Y_k\}$. The following two scheme define the value of a node N as:

$$(\mathbf{Eq_{max}}) \quad N = 1 - \max\{Y_1, \ldots, Y_k\}$$
$$(\mathbf{Eq_{inv}}) \quad N = \prod_{i=1}^{k}(1 - Y_i)$$

Eq_{max} vs. Eq_{inv}

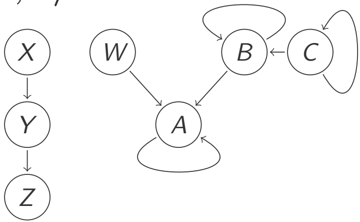
There is a direct correspondence between the solutions to the equations using Eq_{max} and the complete extensions of an abstract argumentation framework.

However, Eq_{max} is not satisfactory for this application because it only takes into account the attacking node(s) with **maximum** value.

Eq_{inv} on the other hand, aggregates the value of all attacking nodes (via multiplication).

Equations according to Eq_{inv}

$\langle \mathbf{S}, \mathbf{R} \rangle$:



Equations according to $\mathbf{Eq_{inv}}$:

$X = 1 \qquad W = 1$

$Y = 1 - X \qquad B = (1 - B)(1 - C)$

$Z = 1 - Y \qquad C = (1 - C)$

$A = (1 - W)(1 - A)(1 - B)$

Solution

in		
X	$=$	1
W	$=$	1
Z	$=$	1

out		
Y	$=$	0
A	$=$	0

und		
B	$=$	$\frac{1}{3}$
C	$=$	$\frac{1}{2}$

This solution recovers the extension $\mathcal{E}$ seen before.

Properties of Eq_{inv}

Theorem. Every solution $\mathbf{f}$ of Eq_{inv} equations written for an argumentation framework yields a complete extension for the network.

Theorem. Every preferred extension of an argumentation framework can be obtained from a solution $\mathbf{f}$ of Eq_{inv} equations written for it.

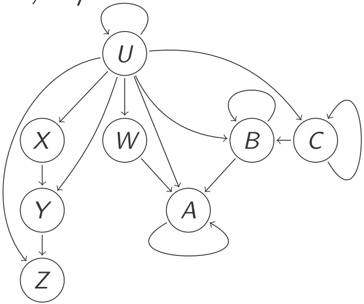
Because of the correspondence, we cannot distinguish between nodes that are **in**, and between the nodes that are **out**. However, note that the undecided nodes have values reflecting the geometry of the network.

In the previous example, $B = \frac{1}{3}$ is arguably more **out** than $C = \frac{1}{2}$.

Obtaining Varying Degrees of **in** and **out**

We simply turn every node into **und**, by adding a new node attacking every node, including itself:

$\langle \mathbf{S'}, \mathbf{R'} \rangle$:



Eq$_{\mathbf{inv}}$:

New Eq_{inv} equations:

$U = 1 - U \quad X = 1 - U$

$Y = (1 - X)(1 - U)$

$Z = (1 - Y)(1 - U)$

$B = (1 - B)(1 - C)(1 - U)$

$C = (1 - C)(1 - U)$

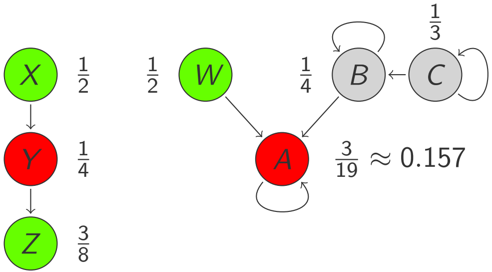
$A = (1 - W)(1 - A)(1 - B)(1 - U)$

SOLUTION: $(U = \frac{1}{2})$

$X = \frac{1}{2} \quad Y = \frac{1}{4} \quad Z = \frac{3}{8} \quad W = \frac{1}{2} \quad A = \frac{3}{19} \quad B = \frac{1}{4} \quad C = \frac{1}{3}$

Effects

$\langle \mathbf{S}, \mathbf{R} \rangle$:



X and W are equally **in**

Z is less **in** than X, W

A is more **out** than Y

B is less **und** than C (alternatively, B is more **out** than C)

Observations

- Mathematically, we are multiplying the right-hand side of all equations by $\frac{1}{2}$
- Since some values are determined in terms of other values the effect is **cumulative** through the chain of attacks
- This correctly reflects the geometry of attacks in the values of the solution, i.e., in width and depth

Observations

- The solutions of the augmented equations have values in the interval $(0, \frac{1}{2}]$
- All values are in a continuum in $(0, \frac{1}{2}]$ and are independent of extensions
- The geometric values can be used to differentiate between the categories **in**, **out** and **und** *relative to an extension*
- More fine-tuned interpretations for extensions are possible but they need to mimic in the network geometry the effect of the choices made for the extension

Conclusions

- The Eq_{inv} equation schema captures the cumulative nature of attacks both in width and depth
- Due to its relationship with traditional semantics Eq_{inv} alone cannot differentiate between nodes that are **in** or **out**.
- By adding a self-attacking node that also attacks all others, we force all nodes into the undecided range, in which nodes **are** differentiated
- This allows for a differentiation of all nodes *relative to an extension*

Future Work

- We conjecture that the solution to the equations of the augmented network is *unique*, but this remains to be proved
- Conceptually, the value $\frac{1}{2}$ makes sense, due to its simplicity and association with the concept of undecidedness
- We are investigating what the use of other values in $(0, 1)$ would mean and its relation to the wider literature context