I heard you the first time: debate in cacophonous surroundings

Paul E. Dunne

Department of Computer Science, University of Liverpool

Overview

- Background
- Modeling approach
- Rudimentary basic solution
- Open issues & further development

Background

Analysis of argument process

1. In general, most work on formal argumentation has been concerned with representing the progression and logical form of argument, e.g. Dung's abstract frameworks and semantics of these via subsets of argument or labelling formalism.
2. In "real" debate, however, there are techniques which may be applied, having no regard for logic or "correctness".
3. While such techniques may indicate inherent weakness in the case being made, on occasion the mere act of using such suffices to force concession of points.

Non-logical argument tactics

Examples

- “stonewalling”, e.g. as examined in Gabbay & Woods (2001)
- prevarication & delaying strategies, e.g. Dunne (2003),
- “malicious” argumentation, e.g. Kuipers & Denzinger (2010)
- “dishonesty” (Sakama, 2012)
- personalised attacks (Budzynska & Reed, 2012)

Non-logical argument tactics

Examples

- "stonewalling", e.g. as examined in Gabbay & Woods (2001)
- prevarication & delaying strategies, e.g. Dunne (2003),
- "malicious" argumentation, e.g. Kuipers & Denzinger (2010)
- "dishonesty" (Sakama, 2012)
- personalised attacks (Budzynska & Reed, 2012)

The methods outlined above all involve features that are *intrinsic to the argument's structure*

Non-logical argument tactics

Examples

- "stonewalling", e.g. as examined in Gabbay & Woods (2001)
- prevarication & delaying strategies, e.g. Dunne (2003),
- "malicious" argumentation, e.g. Kuipers & Denzinger (2010)
- "dishonesty" (Sakama, 2012)
- personalised attacks (Budzynska & Reed, 2012)

The methods outlined above all involve features that are *intrinsic to the argument's structure*

There are, however, other methods which have no dependence whatsoever on the *content* of the argument, let alone its "correctness".

Non-logical argument tactics

Examples

- "stonewalling", e.g. as examined in Gabbay & Woods (2001)
- prevarication & delaying strategies, e.g. Dunne (2003),
- "malicious" argumentation, e.g. Kuipers & Denzinger (2010)
- "dishonesty" (Sakama, 2012)
- personalised attacks (Budzynska & Reed, 2012)

The methods outlined above all involve features that are *intrinsic to the argument's structure*

There are, however, other methods which have no dependence whatsoever on the *content* of the argument, let alone its "correctness".

Such as?

Non-logical argument tactics

Examples

- "stonewalling", e.g. as examined in Gabbay & Woods (2001)
- prevarication & delaying strategies, e.g. Dunne (2003),
- "malicious" argumentation, e.g. Kuipers & Denzinger (2010)
- "dishonesty" (Sakama, 2012)
- personalised attacks (Budzynska & Reed, 2012)

The methods outlined above all involve features that are *intrinsic to the argument's structure*

There are, however, other methods which have no dependence whatsoever on the *content* of the argument, let alone its "correctness".

Such as?

SHOUTING!!!

Raising volume of debate points

Group discussions

Individual **X** makes a point A.

Raising volume of debate points

Group discussions

Individual **X** makes a point A.

Y objects and raises a counter argument B.

Raising volume of debate points

Group discussions

Individual **X** makes a point A.

Y objects and raises a counter argument B.

X unable to think of a response <u>repeats</u> A but <u>at increased volume</u>

Raising volume of debate points

Group discussions

Individual **X** makes a point A.

Y objects and raises a counter argument B.

X unable to think of a response repeats A but at increased volume

Y does the same with B

Raising volume of debate points

Group discussions

Individual **X** makes a point *A*.

Y objects and raises a counter argument *B*.

X unable to think of a response <u>repeats</u> *A* but <u>at increased volume</u>

Y does the same with *B*

etc etc

.....

Raising volume of debate points

Group discussions

Individual **X** makes a point A.

Y objects and raises a counter argument B.

X unable to think of a response repeats A but at increased volume

Y does the same with B

etc etc

.....

Eventually reaching a state where neither A nor B are distinguishable.

Raising volume of debate points

Group discussions

Individual **X** makes a point A.

Y objects and raises a counter argument B.

X unable to think of a response <u>repeats</u> A but <u>at increased volume</u>

Y does the same with B

etc etc

.....

Eventually reaching a state where neither A nor B are distinguishable.

The more participants the greater the likelihood of such behaviour.

Raising volume of debate points

Group discussions

Individual **X** makes a point A.

Y objects and raises a counter argument B.

X unable to think of a response repeats A but at increased volume

Y does the same with B

etc etc

.....

Eventually reaching a state where neither A nor B are distinguishable.

The more participants the greater the likelihood of such behaviour.

Even in **moderated** debates. (eg Parliamentary discussions)

Raising volume of debate points

Group discussions

Individual **X** makes a point A.

Y objects and raises a counter argument B.

X unable to think of a response <u>repeats</u> A but <u>at increased volume</u>

Y does the same with B

etc etc

.....

Eventually reaching a state where neither A nor B are distinguishable.

The more participants the greater the likelihood of such behaviour.

Even in **moderated** debates. (eg Parliamentary discussions)

Especially with "weak" moderators (eg Parliamentary discussions)

Models of debate with noise

Assume we have n participants - $\mathcal{X} = \{x_1, x_2, \ldots, x_n\}$

An argument promoted by x_i may be opposed by that advanced by x_j.

Let $\mathcal{A}$ denote the set of all such opposition: $\mathcal{A} \subset \mathcal{X} \times \mathcal{X}$.

The "attack" $< x_j, x_i >$ has a *volubility* of $\nu(< x_j, x_i >) \in \Re^+$.

Models of debate with noise

Assume we have n participants - $\mathcal{X} \ = \ \{x_1, \ x_2 \ , \ldots , \ x_n\}$

An argument promoted by x_i may be opposed by that advanced by x_j.

Let $\mathcal{A}$ denote the set of all such opposition: $\mathcal{A} \subset \mathcal{X} \times \mathcal{X}$.

The "attack" $< x_j, x_i >$ has a *volubility* of $\nu(< x_j, x_i >) \in \Re^+$.

The structure $(< \mathcal{X}, \mathcal{A} >, \nu \ : \ \mathcal{A} \rightarrow \Re^+)$ is a (weighted) Dung AF.

Models of debate with noise

Assume we have n participants - $\mathcal{X} = \{x_1, x_2, \ldots, x_n\}$

An argument promoted by x_i may be opposed by that advanced by x_j.

Let $\mathcal{A}$ denote the set of all such opposition: $\mathcal{A} \subset \mathcal{X} \times \mathcal{X}$.

The "attack" $< x_j, x_i >$ has a *volubility* of $\nu(< x_j, x_i >) \in \Re^+$.

The structure $(< \mathcal{X}, \mathcal{A} >, \nu : \mathcal{A} \to \Re^+)$ is a (weighted) Dung AF.

Questions

Q1. Assuming the debate described by $(< \mathcal{X}, \mathcal{A} >, \nu)$ has an (external) moderator, M, how may M determine if $(< \mathcal{X}, \mathcal{A} >, \nu)$ is "unstable" by reason of $\nu(< x_i, x_j >)$ being "too loud"?

Models of debate with noise

Assume we have n participants - $\mathcal{X} = \{x_1, x_2, \ldots, x_n\}$

An argument promoted by x_i may be opposed by that advanced by x_j.

Let $\mathcal{A}$ denote the set of all such opposition: $\mathcal{A} \subset \mathcal{X} \times \mathcal{X}$.

The "attack" $< x_j, x_i >$ has a *volubility* of $\nu(< x_j, x_i >) \in \Re^+$.

The structure $(< \mathcal{X}, \mathcal{A} >, \nu : \mathcal{A} \to \Re^+)$ is a (weighted) Dung AF.

Questions

Q1. Assuming the debate described by $(< \mathcal{X}, \mathcal{A} >, \nu)$ has an (external) moderator, M, how may M determine if $(< \mathcal{X}, \mathcal{A} >, \nu)$ is "unstable" by reason of $\nu(< x_i, x_j >)$ being "too loud"?

Q2. In this event what actions can M take to "restore order"?

Models of debate with noise

Assume we have n participants - $\mathcal{X} = \{x_1, x_2, \ldots, x_n\}$

An argument promoted by x_i may be opposed by that advanced by x_j.

Let $\mathcal{A}$ denote the set of all such opposition: $\mathcal{A} \subset \mathcal{X} \times \mathcal{X}$.

The "attack" $< x_j, x_i >$ has a *volubility* of $\nu(< x_j, x_i >) \in \Re^+$.

The structure $(< \mathcal{X}, \mathcal{A} >, \nu : \mathcal{A} \rightarrow \Re^+)$ is a (weighted) Dung AF.

Questions

Q1. Assuming the debate described by $(< \mathcal{X}, \mathcal{A} >, \nu)$ has an (external) moderator, M, how may M determine if $(< \mathcal{X}, \mathcal{A} >, \nu)$ is "unstable" by reason of $\nu(< x_i, x_j >)$ being "too loud"?

Q2. In this event what actions can M take to "restore order"?

We focus on Q1 in this talk, discussing the second question briefly later.

Debate evolution

At any given point the structure $\mathcal{D} = (< \mathcal{X}, \mathcal{A} >, \nu)$ will be fixed.

As discussion develops, however, $\nu(< x_i, x_j >)$ may vary in intensity.

This leads to the main structure of interest being the <u>sequence</u>

$$\underline{\mathcal{D}} \;=\; < \mathcal{D}_0, \mathcal{D}_1, \ldots, \mathcal{D}_k, \ldots >$$

Debate evolution

At any given point the structure $\mathcal{D} = (< \mathcal{X}, \mathcal{A} >, \nu)$ will be fixed.

As discussion develops, however, $\nu(< x_i, x_j >)$ may vary in intensity.

This leads to the main structure of interest being the sequence

$$\underline{\mathcal{D}} \;=\; < \mathcal{D}_0, \mathcal{D}_1, \ldots, \mathcal{D}_k, \ldots >$$

This may be infinite.

Debate evolution

At any given point the structure $\mathcal{D} = (< \mathcal{X}, \mathcal{A} >, \nu)$ will be fixed.

As discussion develops, however, $\nu(< x_i, x_j >)$ may vary in intensity.

This leads to the main structure of interest being the sequence

$$\underline{\mathcal{D}} \;=\; < \mathcal{D}_0, \mathcal{D}_1, \ldots, \mathcal{D}_k, \ldots >$$

This may be infinite.

As the sequence progresses we assume $\mathcal{X}$ remains fixed, but

1. $\mathcal{A}_k \subseteq \mathcal{A}_{k-1}$: "new" attacks do **not** appear.
2. $\nu_k \;:\; \mathcal{A}_0 \to \Re^+ \cup \{0\}$.
3. $< x_i, x_j > \notin \mathcal{A}_k$ if $\nu_k(< x_i, x_j >) = 0$.

Debate evolution

At any given point the structure $\mathcal{D} = (<\mathcal{X}, \mathcal{A}>, \nu)$ will be fixed.

As discussion develops, however, $\nu(<x_i, x_j>)$ may vary in intensity.

This leads to the main structure of interest being the <u>sequence</u>

$$\underline{\mathcal{D}} \ = \ <\mathcal{D}_0, \mathcal{D}_1, \ldots, \mathcal{D}_k, \ldots>$$

This may be <u>infinite</u>.

As the sequence progresses we assume $\mathcal{X}$ remains fixed, but

1. $\mathcal{A}_k \subseteq \mathcal{A}_{k-1}$: “new” attacks do **not** appear.
2. $\nu_k \ : \ \mathcal{A}_0 \to \Re^+ \cup \{0\}$.
3. $<x_i, x_j> \notin \mathcal{A}_k$ if $\nu_k(<x_i, x_j>) = 0$.
   Notice this means attacks may be (permanently) retracted, eg should $\nu_{k-1}(<x_i, x_j>) > 0$ and $\nu_k(<x_i, x_j>) = 0$.

Debate evolution

At any given point the structure $\mathcal{D} = (< \mathcal{X}, \mathcal{A} >, \nu)$ will be fixed.

As discussion develops, however, $\nu(< x_i, x_j >)$ may vary in intensity.

This leads to the main structure of interest being the <u>sequence</u>

$$\underline{\mathcal{D}} \;=\; < \mathcal{D}_0, \mathcal{D}_1, \ldots, \mathcal{D}_k, \ldots >$$

This may be <u>infinite</u>.

As the sequence progresses we assume $\mathcal{X}$ remains fixed, but

1. $\mathcal{A}_k \subseteq \mathcal{A}_{k-1}$: "new" attacks do **not** appear.
2. $\nu_k \;:\; \mathcal{A}_0 \to \Re^+ \cup \{0\}$.
3. $< x_i, x_j > \notin \mathcal{A}_k$ if $\nu_k(< x_i, x_j >) = 0$.
   Notice this means attacks may be (permanently) retracted, eg should $\nu_{k-1}(< x_i, x_j >) > 0$ and $\nu_k(< x_i, x_j >) = 0$.

Q1 concerns deciding if ν_k is such that M ought to intervene.

Initial solution & assumptions

The champion of x_i pushes it with an overall **stridency**, S_i.

An attack $< x_i, x_j >$ is promoted with **volubility**, $\nu(< x_i, x_j >)$.

The extent to which x_i "disrupts" x_j is F_{ji} and is $\nu(< x_i, x_j >)$.

Initial solution & assumptions

The champion of x_i pushes it with an overall **stridency**, S_i.

An attack $< x_i, x_j >$ is promoted with **volubility**, $\nu(< x_i, x_j >)$.

The extent to which x_i "disrupts" x_j is F_{ji} and is $\nu(< x_i, x_j >)$.

The agent promoting x_i is prepared to tolerate some **maximum** level of interference, μ_i.

Initial solution & assumptions

The champion of x_i pushes it with an overall **stridency**, S_i.

An attack $< x_i, x_j >$ is promoted with **volubility**, $\nu(< x_i, x_j >)$.

The extent to which x_i "disrupts" x_j is F_{ji} and is $\nu(< x_i, x_j >)$.

The agent promoting x_i is prepared to tolerate some **maximum** level of interference, μ_i.

This could be set by M or decided by x_i.

Initial solution & assumptions

The champion of x_i pushes it with an overall **stridency**, S_i.

An attack $< x_i, x_j >$ is promoted with **volubility**, $\nu(< x_i, x_j >)$.

The extent to which x_i "disrupts" x_j is F_{ji} and is $\nu(< x_i, x_j >)$.

The agent promoting x_i is prepared to tolerate some **maximum** level of interference, μ_i.

This could be set by M or decided by x_i.

Instability

The debate, $\mathcal{D}$, is in an **unstable** state should the **total** interference that any agent, i, has to contend with exceed the maximum level, μ_i, it is prepared to tolerate.

Recognising unstable debates

Suppose that,

$$\underline{\mu} =< \mu_1, \mu_2, \ldots, \mu_n >$$

Recognising unstable debates

Suppose that,

$$\underline{\mu} =< \mu_1, \mu_2, \ldots, \mu_n >$$

Consider the **ratio**,

$$\frac{S_i}{\sum_{j \neq i} F_{ij} \times S_j}$$

Informally, the amount by which i "out shouts" the noise directed against x_i where this volume is considered relative to the noise emanating from j in pushing x_j.

Recognising unstable debates

Suppose that,

$$\underline{\mu} =< \mu_1, \mu_2, \ldots, \mu_n >$$

Consider the **ratio**,

$$\frac{S_i}{\sum_{j \neq i} F_{ij} \times S_j}$$

Informally, the amount by which i "out shouts" the noise directed against x_i where this volume is considered relative to the noise emanating from j in pushing x_j.

Then acceptable levels of noise for **every** agent requires,

$$\forall\ 1 \leq i \leq n \qquad \left(\frac{S_i}{\sum_{j \neq i} F_{ij} \times S_j} \right) \leq \mu_i \tag{1}$$

Matrix Interpretation

Consider now the $n \times n$ matrices, **F** of force and **C** of constraint given as,

$$\mathbf{F}_{ij} = \begin{cases} 0 & \text{if } i = j \\ F_{ij} & \text{otherwise} \end{cases} \quad ; \quad \mathbf{C}_{ij} = \begin{cases} \mu_i & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

Matrix Interpretation

Consider now the $n \times n$ matrices, **F** of force and **C** of constraint given as,

$$\mathbf{F}_{ij} = \begin{cases} 0 & \text{if } i = j \\ F_{ij} & \text{otherwise} \end{cases} \quad ; \quad \mathbf{C}_{ij} = \begin{cases} \mu_i & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

Furthermore, denote by $\underline{\mathbf{S}}$ the **c**olumn vector,

$$\underline{\mathbf{S}} = \begin{pmatrix} S_1 \\ \cdot \\ \cdot \\ S_n \end{pmatrix}$$

Matrix Interpretation

Consider now the $n \times n$ matrices, **F** of <u>force</u> and **C** of <u>constraint</u> given as,

$$\mathbf{F}_{ij} = \begin{cases} 0 & \text{if } i = j \\ F_{ij} & \text{otherwise} \end{cases} \quad ; \quad \mathbf{C}_{ij} = \begin{cases} \mu_i & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

Furthermore, denote by $\underline{\mathbf{S}}$ the **c**olumn vector,

$$\underline{\mathbf{S}} = \begin{pmatrix} S_1 \\ \cdot \\ \cdot \\ S_n \end{pmatrix}$$

Then (writing **B** for $\mathbf{C} \times \mathbf{F}$) $\mathcal{D}$ satisfies (1) if, component-wise,

$$\mathbf{B} \times \underline{\mathbf{S}} \geq \underline{\mathbf{S}} \quad (2)$$

In which eigenvalues and eigenvectors enter

The elements defining **C** and **F** are given: what will be tolerated as noise (μ_i) and the (current) levels of volubility.

In which eigenvalues and eigenvectors enter

The elements defining **C** and **F** are given: what will be tolerated as noise (μ_i) and the (current) levels of volubility.

Putting aside for one moment how stridency (S_i) will be related to volubility, (2) allows M to set (and monitor) **S**, **provided that B is well-behaved**.

In which eigenvalues and eigenvectors enter

The elements defining **C** and **F** are given: what will be tolerated as noise (μ_i) and the (current) levels of volubility.

Putting aside for one moment how stridency (S_i) will be related to volubility, (2) allows M to set (and monitor) **S**, **provided that B is well-behaved**.

What does "well-behaved" mean?

That **B** has an eigenvalue λ with associated eigenvector, $\underline{z}$ satisfying,

a. $\lambda \geq 1$.

b. $\underline{z}$ is positive, ie $z_i > 0$ for each $1 \leq i \leq n$.

In which eigenvalues and eigenvectors enter

The elements defining **C** and **F** are given: what will be tolerated as noise (μ_i) and the (current) levels of volubility.

Putting aside for one moment how stridency (S_i) will be related to volubility, (2) allows M to set (and monitor) $\underline{\mathbf{S}}$, **provided that B is well-behaved**.

What does "well-behaved" mean?

That **B** has an eigenvalue λ with associated eigenvector, $\underline{z}$ satisfying,

a. $\lambda \geq 1$.

b. $\underline{z}$ is positive, ie $z_i > 0$ for each $1 \leq i \leq n$.

so ensuring

$$\mathbf{B} \times \underline{z} \quad = \quad \lambda \underline{z} \quad \geq \quad \underline{z}$$

“Well-behaved” systems

The conditions prescribed allow M to fix stridency levels, S_i, for each participant in such a way that should one player deviate then the resulting $\mathcal{D}$ is seen as **unstable**.

"Well-behaved" systems

The conditions prescribed allow M to fix stridency levels, S_i, for each participant in such a way that should one player deviate then the resulting $\mathcal{D}$ is seen as **unstable**.

2 Questions

"Well-behaved" systems

The conditions prescribed allow M to fix stridency levels, S_i, for each participant in such a way that should one player deviate then the resulting $\mathcal{D}$ is seen as **unstable**.

2 Questions

A. Which configurations $(<\mathcal{X},\mathcal{A}>,\nu)$ and $\underline{\mu}$ are "well-behaved"?

“Well-behaved” systems

The conditions prescribed allow M to fix stridency levels, S_i, for each participant in such a way that should one player deviate then the resulting $\mathcal{D}$ is seen as **unstable**.

2 Questions

A. Which configurations $(< \mathcal{X}, \mathcal{A} >, \nu)$ and $\underline{\mu}$ are “well-behaved”?

B. How ought *stridency* to be related to *volubility*?

"Well-behaved" systems

The conditions prescribed allow M to fix stridency levels, S_i, for each participant in such a way that should one player deviate then the resulting $\mathcal{D}$ is seen as **unstable**.

2 Questions

A. Which configurations $(<\mathcal{X},\mathcal{A}>,\nu)$ and $\underline{\mu}$ are "well-behaved"?

B. How ought *stridency* to be related to *volubility*?

Question A: a sufficient condition for a system to be well-behaved

If $<\mathcal{X},\mathcal{A}>$ is **strongly-connected** then $\mathbf{B}=\mathbf{C}\times\mathbf{F}$ has a unique maximum eigenvalue $\lambda>0$ with associated positive eigenvector $\underline{z}$.

"Well-behaved" systems

The conditions prescribed allow M to fix stridency levels, S_i, for each participant in such a way that should one player deviate then the resulting $\mathcal{D}$ is seen as **unstable**.

2 Questions

A. Which configurations $(<\mathcal{X},\mathcal{A}>,\nu)$ and $\underline{\mu}$ are "well-behaved"?

B. How ought *stridency* to be related to *volubility*?

Question A: a sufficient condition for a system to be well-behaved

If $<\mathcal{X},\mathcal{A}>$ is **strongly-connected** then $\mathbf{B} = \mathbf{C} \times \mathbf{F}$ has a unique maximum eigenvalue $\lambda > 0$ with associated positive eigenvector $\underline{z}$.

Hence **B** is well-behaved in the sense above if $\lambda \geq 1$.

Question B: Stridency & Volubility

Simple Approach

Since each agent is already promoting its argument against others (via attacks $< x_i, x_j > \in \mathcal{A}$) simply set its **current** level of stridency to be

Question B: Stridency & Volubility

Simple Approach

Since each agent is already promoting its argument against others (via attacks $< x_i, x_j > \in \mathcal{A}$) simply set its **current** level of stridency to be

$$S_i = \sum_{<x_i,x_j> \in \mathcal{A}} \nu(< x_i, x_j >)$$

Question B: Stridency & Volubility

Simple Approach

Since each agent is already promoting its argument against others (via attacks $< x_i, x_j > \in \mathcal{A}$) simply set its **current** level of stridency to be

$$S_i = \sum_{<x_i,x_j>\in\mathcal{A}} \nu(< x_i, x_j >)$$

Note: this is just **one** option

No restrictions on how S_i is distributed over $< x_i, x_j > \in \mathcal{A}$.

Question B: Stridency & Volubility

Simple Approach

Since each agent is already promoting its argument against others (via attacks $< x_i, x_j > \in \mathcal{A}$) simply set its **current** level of stridency to be

$$S_i \quad = \quad \sum_{<x_i,x_j>\in\mathcal{A}} \nu(< x_i, x_j >)$$

Note: this is just **one** option

No restrictions on how S_i is distributed over $< x_i, x_j > \in \mathcal{A}$.

Consequences: new form of **F**

$$S_i \quad = \quad \sum_{<x_i,x_j>\in\mathcal{A}} \nu(< x_i, x_j >) \quad = \quad \sum_{j \neq i} F_{ji}$$

What happens with revised form of $\mathbf{F}$?

Recall the transpose of $\mathbf{F}$ is the $n \times n$ matrix, $\mathbf{F}^T$ with $\mathbf{F}^T_{ij} = \mathbf{F}_{ji}$.
The relevant "stridency-to-interference" condition is now

What happens with revised form of $\mathbf{F}$?

Recall the transpose of $\mathbf{F}$ is the $n \times n$ matrix, $\mathbf{F}^T$ with $\mathbf{F}^T_{ij} \;=\; \mathbf{F}_{ji}$.
The relevant "stridency-to-interference" condition is now

$$\forall\, 1 \leq i \leq n \quad \left(\frac{\sum_{j \neq i} \; F_{ji}}{\sum_{j \neq i} \; (F_{ij} \times \sum_{k \neq j} \; F_{kj})} \right) \quad \leq \quad \mu_i \tag{3}$$

What happens with revised form of $\mathbf{F}$?

Recall the transpose of $\mathbf{F}$ is the $n \times n$ matrix, $\mathbf{F}^T$ with $\mathbf{F}^T_{ij} = \mathbf{F}_{ji}$.
The relevant "stridency-to-interference" condition is now

$$\forall\ 1 \leq i \leq n \quad \left(\frac{\sum_{j \neq i} F_{ji}}{\sum_{j \neq i} \left(F_{ij} \times \sum_{k \neq j} F_{kj} \right)} \right) \leq \mu_i \tag{3}$$

Letting $\underline{\mathbf{1}}$ be the $n \times 1$ column vector of 1s, after some manipulation we now obtain,

What happens with revised form of $\mathbf{F}$?

Recall the <u>transpose</u> of $\mathbf{F}$ is the $n \times n$ matrix, $\mathbf{F}^T$ with $\mathbf{F}^T_{ij} = \mathbf{F}_{ji}$.
The relevant "stridency-to-interference" condition is now

$$\forall\, 1 \leq i \leq n \quad \left(\frac{\sum_{j \neq i} F_{ji}}{\sum_{j \neq i} \left(F_{ij} \times \sum_{k \neq j} F_{kj} \right)} \right) \leq \mu_i \tag{3}$$

Letting $\underline{\mathbf{1}}$ be the $n \times 1$ column vector of 1s, after some manipulation we now obtain,

For stridency as total volubility

$\mathcal{D}$ is stable wrt $\underline{\mu}$ if

There is an eigenvalue λ of $\mathbf{C} \times \mathbf{F}$ with $\lambda \geq 1$.

AND

$\mathbf{F}^T \times \underline{\mathbf{1}}$ is an eigenvector of λ.

Summary & Open Issues

The models and analysis are *very preliminary* and there are a number of issues of interest which we, very briefly, summarise.

Summary & Open Issues

The models and analysis are *very preliminary* and there are a number of issues of interest which we, very briefly, summarise.

Suppose $(\mathcal{D}, \underline{\mu})$ is unstable, what actions can M take?

Summary & Open Issues

The models and analysis are *very preliminary* and there are a number of issues of interest which we, very briefly, summarise.

Suppose $(\mathcal{D}, \underline{\mu})$ is unstable, what actions can M take?

a. Adjust $\underline{\mu}$? Could be seen as "weakness" on M's part (changing tolerance rather than dealing with stridency).

Summary & Open Issues

The models and analysis are *very preliminary* and there are a number of issues of interest which we, very briefly, summarise.

Suppose $(\mathcal{D}, \underline{\mu})$ is unstable, what actions can M take?

a. Adjust $\underline{\mu}$? Could be seen as "weakness" on M's part (changing tolerance rather than dealing with stridency).
b. Expel "over-loud" agents? How to choose these?

   One at a time or collectively?
   What if the resulting graph form ceases to be strongly-connected?
   Is expulsion temporary? What conditions allow "readmission"?

Summary & Open Issues

The models and analysis are *very preliminary* and there are a number of issues of interest which we, very briefly, summarise.

Suppose $(\mathcal{D}, \underline{\mu})$ is unstable, what actions can M take?

a. Adjust $\underline{\mu}$? Could be seen as "weakness" on M's part (changing tolerance rather than dealing with stridency).
b. Expel "over-loud" agents? How to choose these?
    - One at a time or collectively?
    - What if the resulting graph form ceases to be strongly-connected?
    - Is expulsion temporary? What conditions allow "readmission"?

Other choices for relating stridency and volubility?

Summary & Open Issues

The models and analysis are *very preliminary* and there are a number of issues of interest which we, very briefly, summarise.

Suppose $(\mathcal{D}, \underline{\mu})$ is unstable, what actions can M take?

a. Adjust $\underline{\mu}$? Could be seen as "weakness" on M's part (changing tolerance rather than dealing with stridency).
b. Expel "over-loud" agents? How to choose these?

    One at a time or collectively?
    What if the resulting graph form ceases to be strongly-connected?
    Is expulsion temporary? What conditions allow "readmission"?

Other choices for relating stridency and volubility?

Strong-connectivity is **one** sufficient condition for analysis. Others?