On the Acceptability Semantics of Argumentation Frameworks with Recursive Attack and Support

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Introduction

- In the last decade the study of argumentation systems with support relations has greatly increased.
- Several interpretations for the notion of support were proposed in the literature (e.g., general, evidential, deductive, necessary, etc.).
- For each interpretation of support an Abstract Bipolar Argumentation Framework with attack and support relations can be defined.

Introduction

- Another prominent line of work started with the consideration of high level attacks (e.g., EAF, AFRA).
- The combination of these two led to the characterization of recursive attack and support relations.
- In [CGGS15] the Attack-Support Argumentation Framework (ASAF) was proposed, allowing for attack and support for arguments as well as the attack and support relations at any level.

[CGGS15] Andrea Cohen, Sebastian Gottifredi, Alejandro J. García and Guillermo R. Simari: An approach to abstract argumentation with recursive attack and support. Journal of Applied Logic 13(4): 509-533 (2015).

Introduction

- Acceptability calculus in [CGGS15] is handled by translating the ASAF into a Dung's AF.
- In this work we characterize the acceptability semantics of the ASAF following an extensional approach:
  - Complete semantics
  - Preferred semantics
  - Stable semantics
  - Grounded semantics
- We show that our characterization satisfies different properties identified for Dung's Abstract Argumentation Frameworks (AFs); and other properties related to the nature of the attack and support relations of the ASAF.

Outline

- Attack-Support Argumentation Framework (ASAF)
- Defeats in the ASAF
  - Unconditional Defeats (support-independent)
  - Conditional Defeats (support-dependent)
- Acceptability Semantics of the ASAF
  - Basic Semantic Notions
  - Extensions
- Formal Results

Attack-Support Argumentation Framework (ASAF)

ASAF - Attack-Support Argumentation Framework

- An Attack-Support Argumentation Framework (ASAF) is a tuple $\langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$, where:
  - $\mathbb{A}$ is a set of arguments
  - $\mathbb{R} \subseteq \mathbb{A} \times (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ is an attack relation
  - $\mathbb{S} \subseteq \mathbb{A} \times (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ is a necessary support relation
  - $\mathbb{S}$ is acyclic
  - $\mathbb{R} \cap \mathbb{S} = \emptyset$

- Graphically:
  - $\alpha = (a, b) \in \mathbb{R}$ is represented by $a \xrightarrow{\alpha} b$
  - $\beta = (c, d) \in \mathbb{S}$ is represented by $c \xRightarrow{\beta} d$

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Given $(a, b) \in \mathbb{S}$
If b is accepted, then a is also accepted
(if a is not accepted, then b is not accepted either)

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Given $\alpha = (a, b) \in \mathbb{R}$, $\beta = (c, d) \in \mathbb{S}$:
$src(\alpha) = a$; $src(\beta) = c$
$trg(\alpha) = b$; $trg(\beta) = d$

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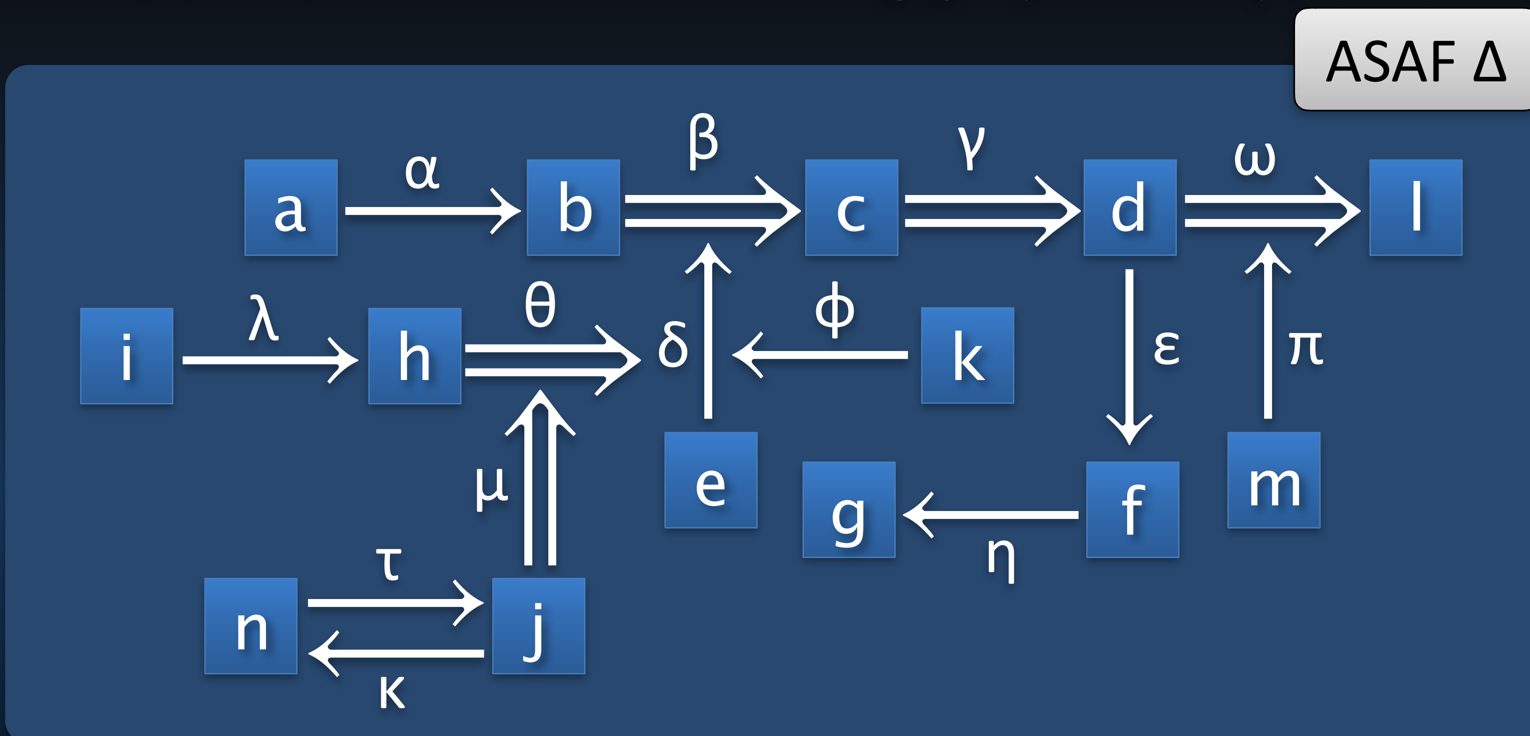
ASAF - Example

- Let Δ be an ASAF with the following graphical representation:



- First-level interactions: α = (a,b), ε = (d,f), η = (f,g), λ = (i,h), τ = (n,j), κ = (j,n), β = (b,c), γ = (c,d) and ω = (d,l).

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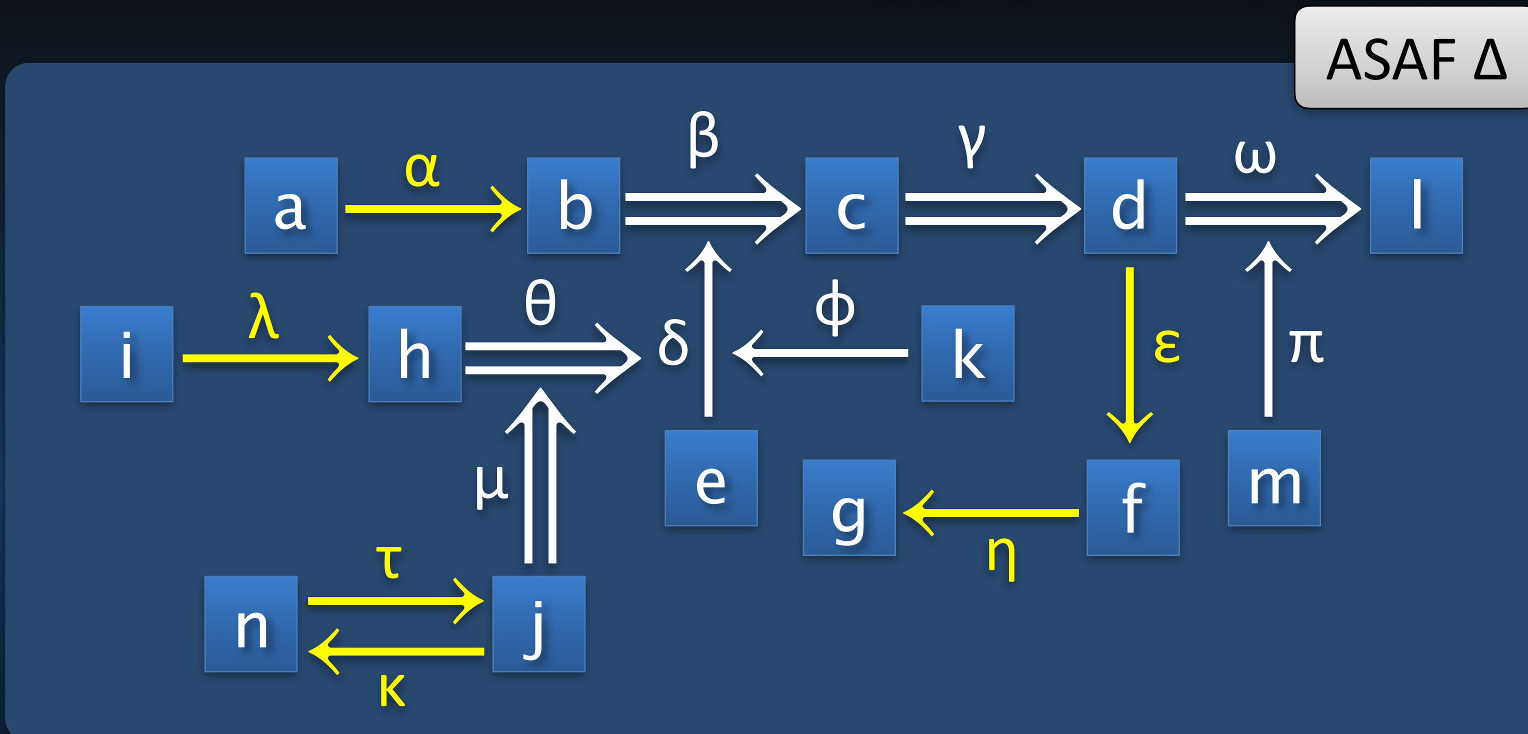

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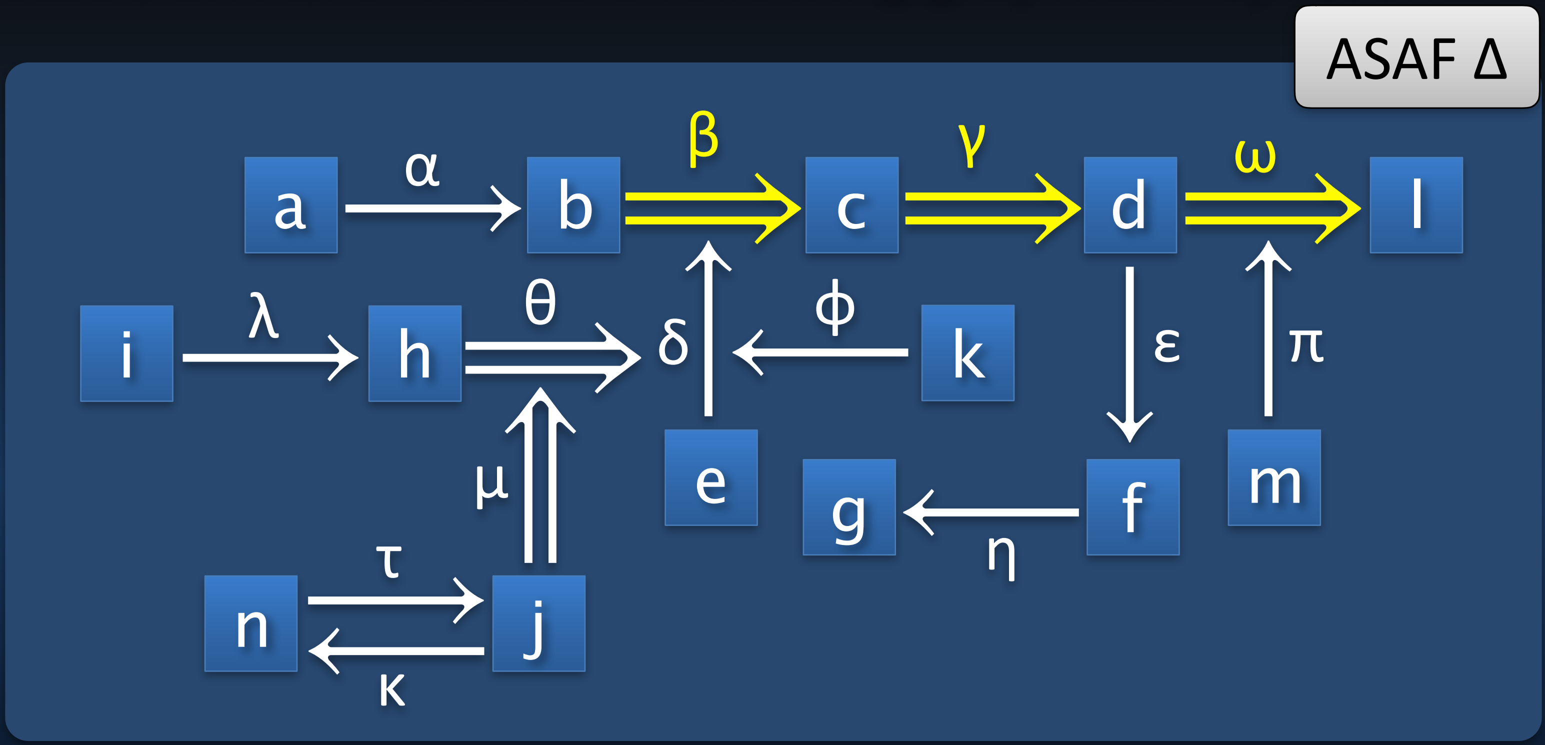

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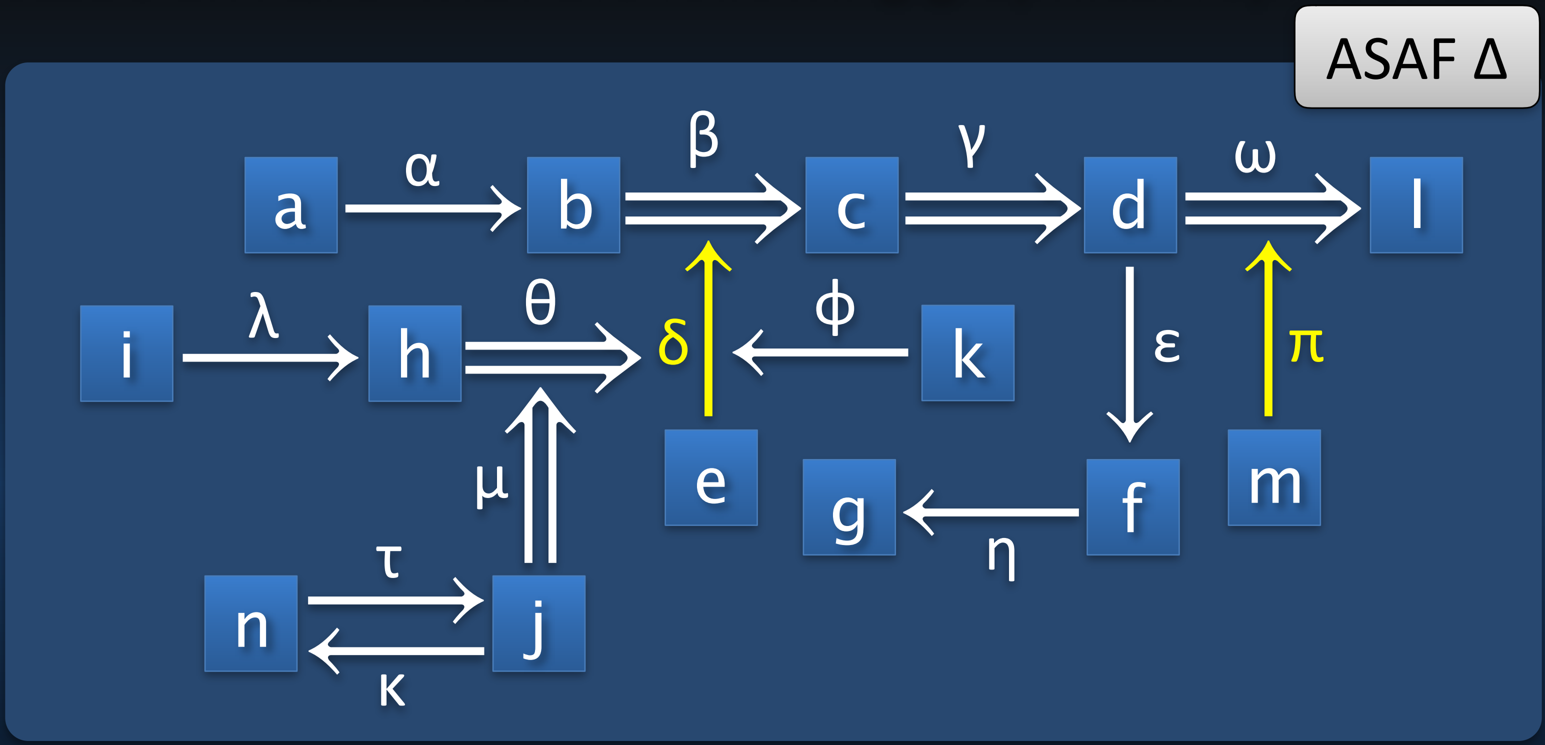

- Second-level interactions: δ = (e,β), π = (m,ω).

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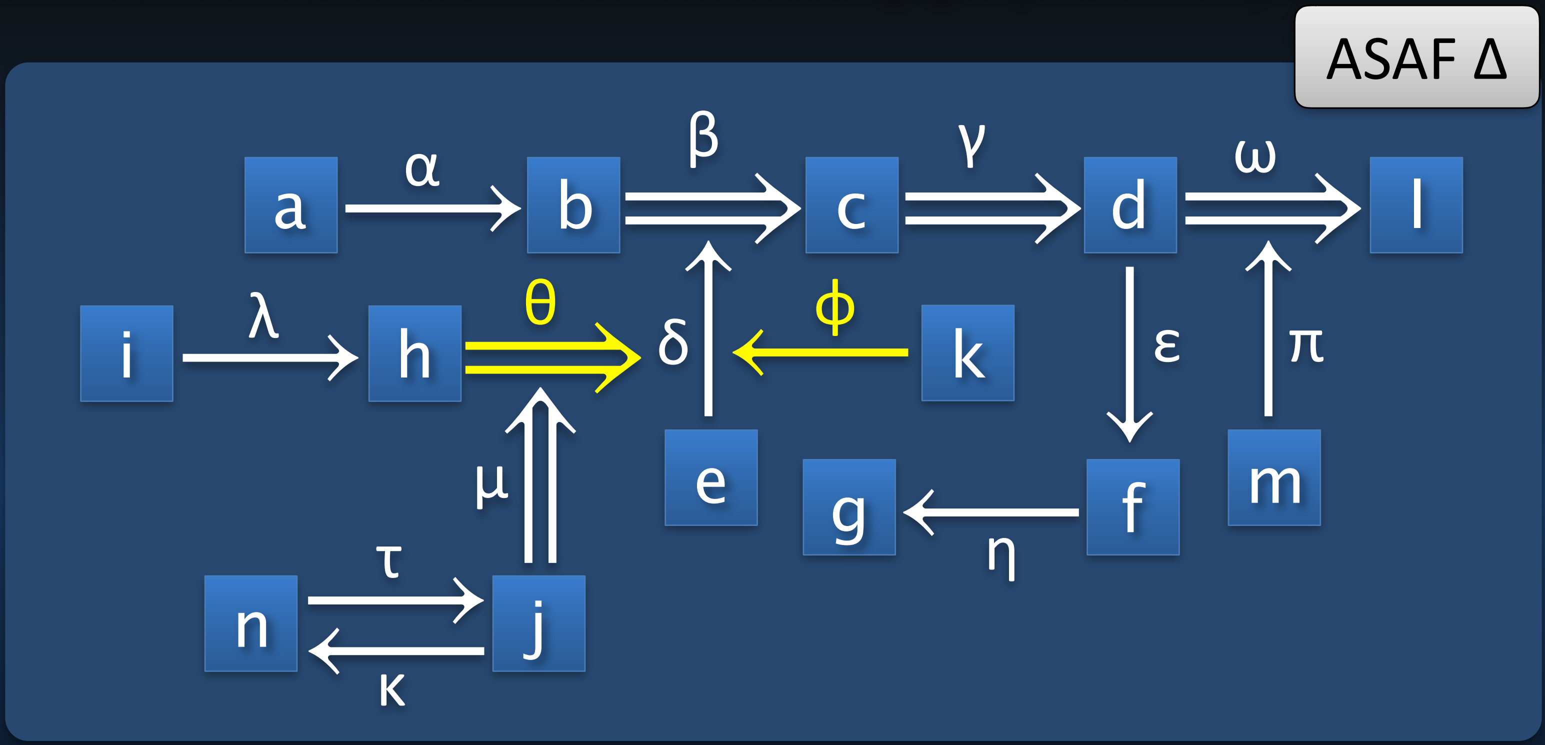

- Third-level interactions: $\phi = (k,\delta)$ and $\theta = (h,\delta)$.

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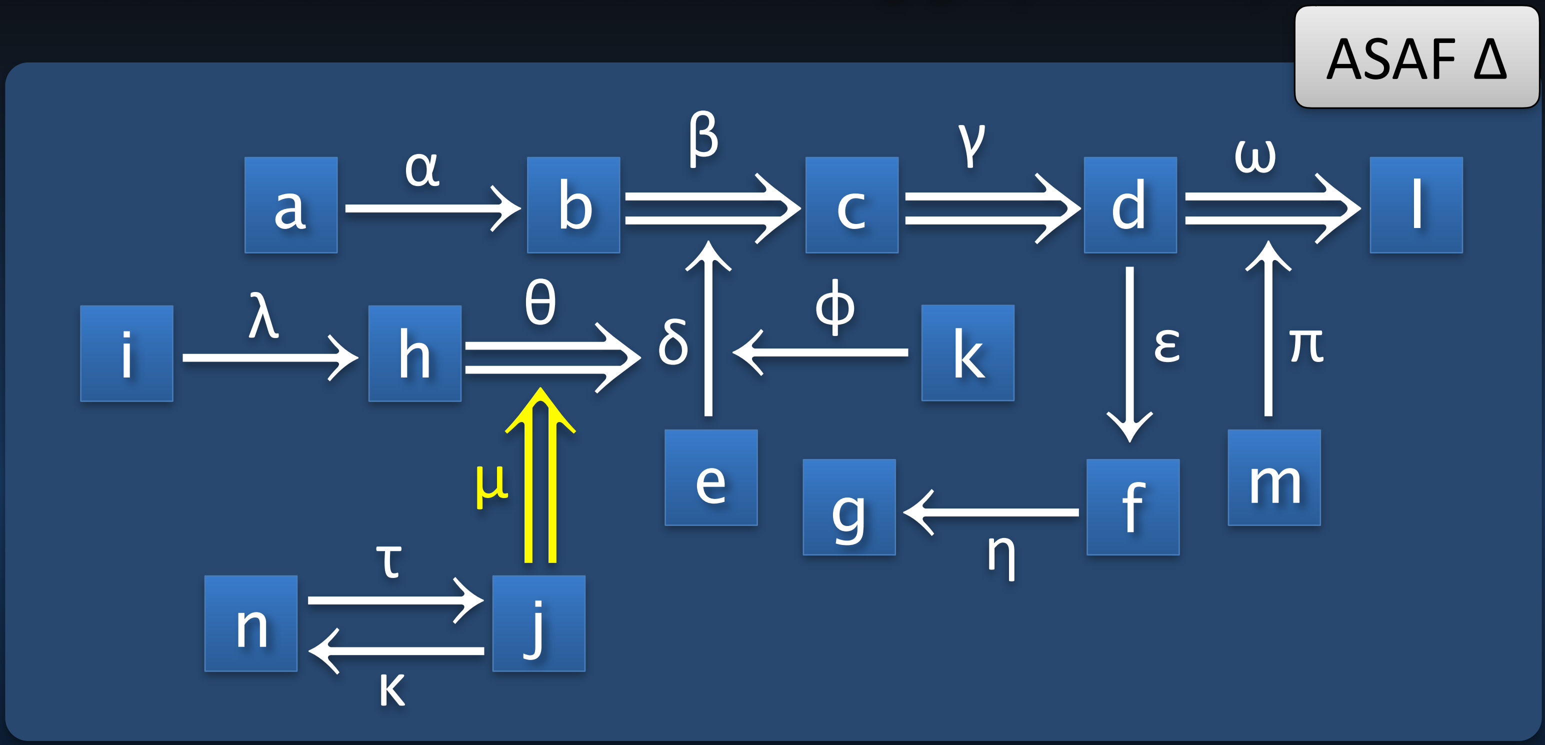

- Fourth-level interactions: μ = (j,θ).

Defeats in ASAF
Conditional + Unconditional

ASAF - Defeats

- We regard attacks as the subjects able to defeat arguments, attacks or supports.
  - Unconditional Defeats: are inferred directly from the attack relation.
  - Conditional Defeats: are inferred from the combination of the attack and support relations.

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ASAF - Unconditional Defeats

- Given an ASAF $\langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ we identify two cases in which unconditional defeats occur:
  - Direct Defeat: Let $\alpha \in \mathbb{R}$ and $X \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$.
    α d-def X if $trg(\alpha) = X$.
  - Indirect Defeat: Let $\alpha, \beta \in \mathbb{R}$.
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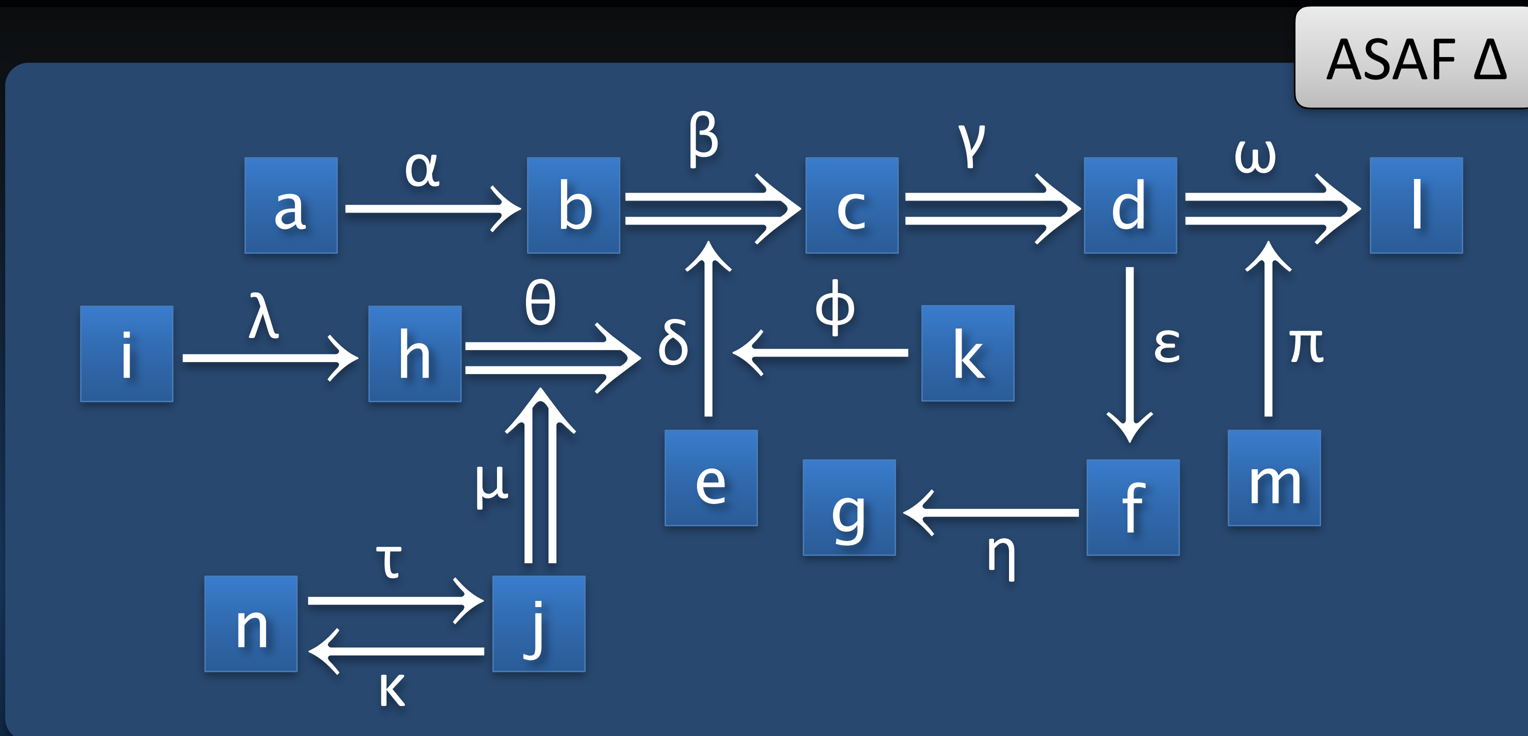

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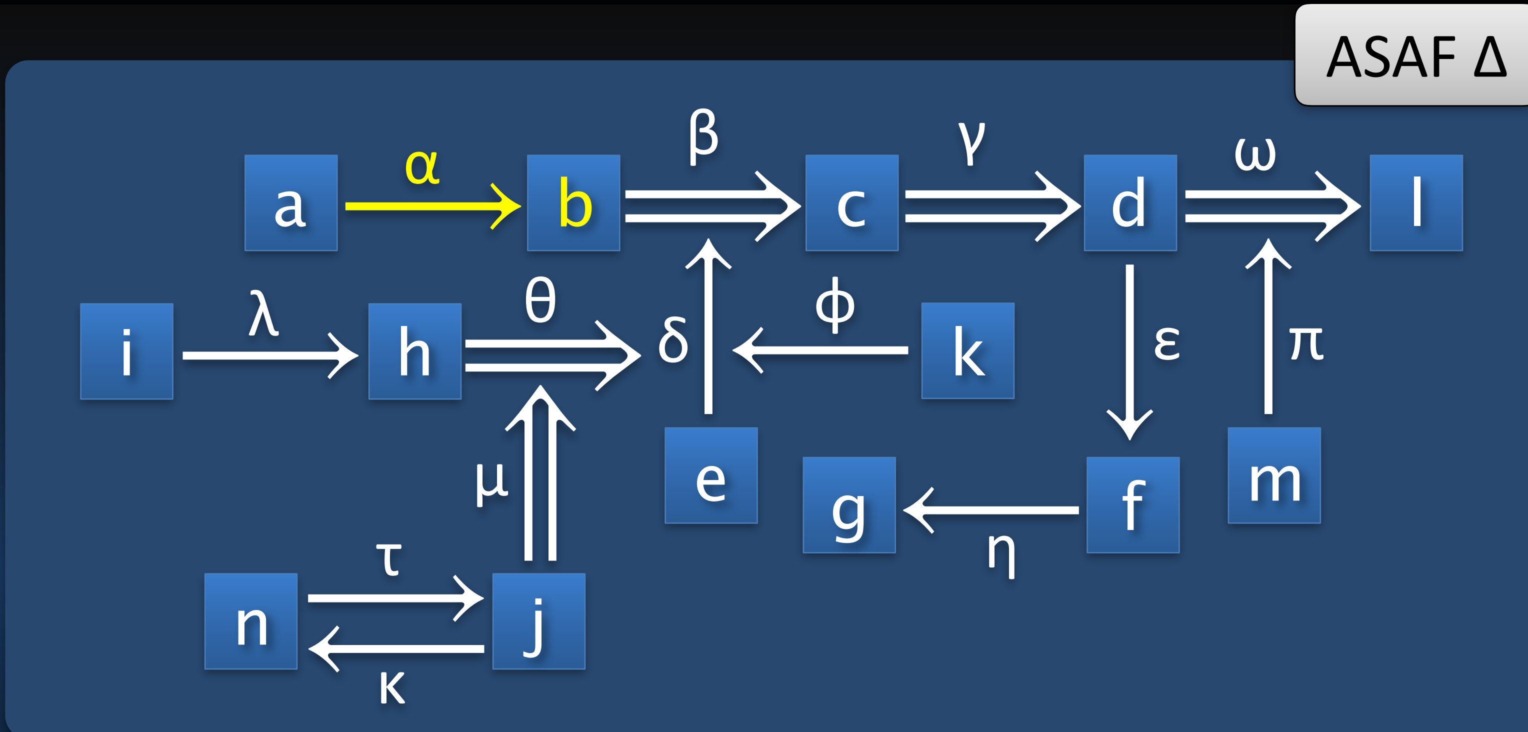

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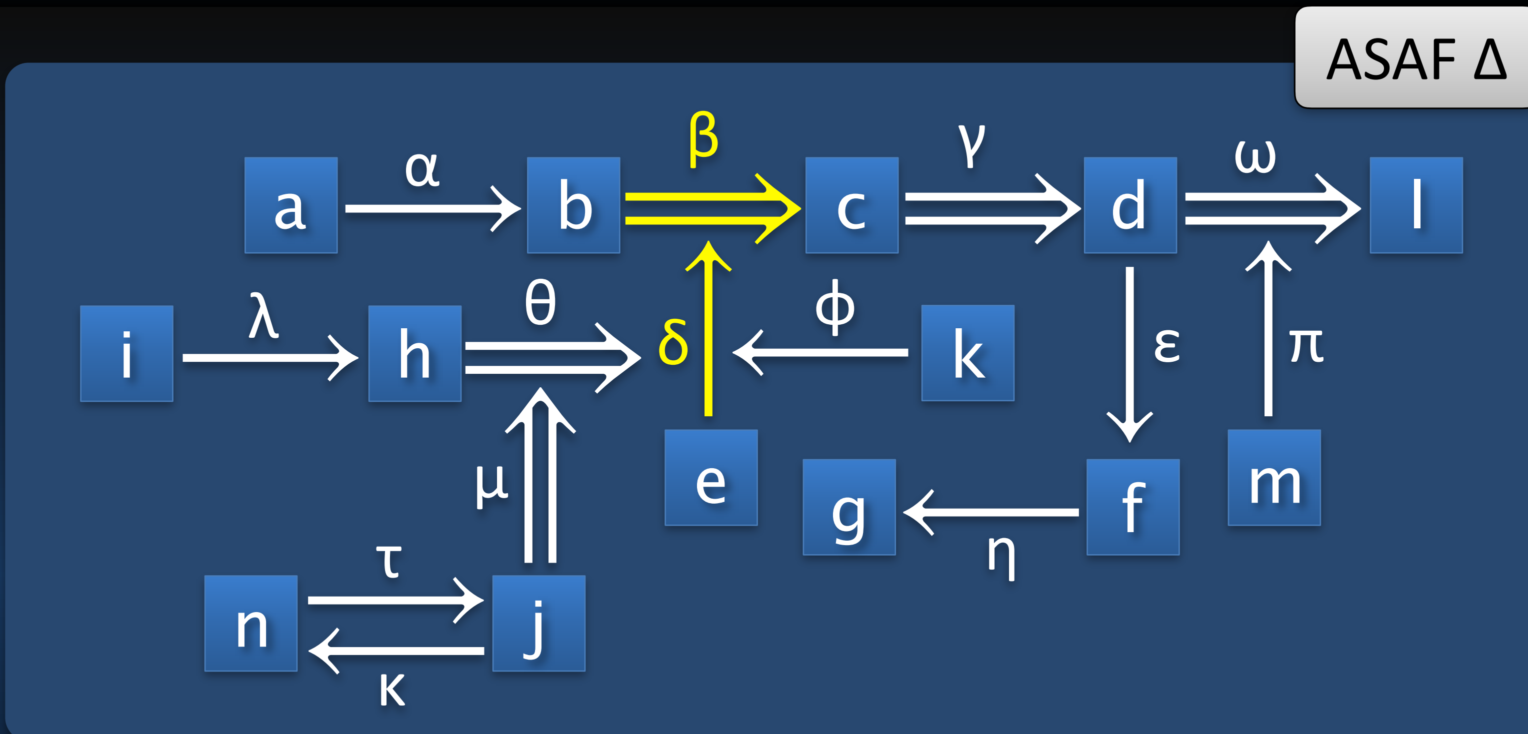

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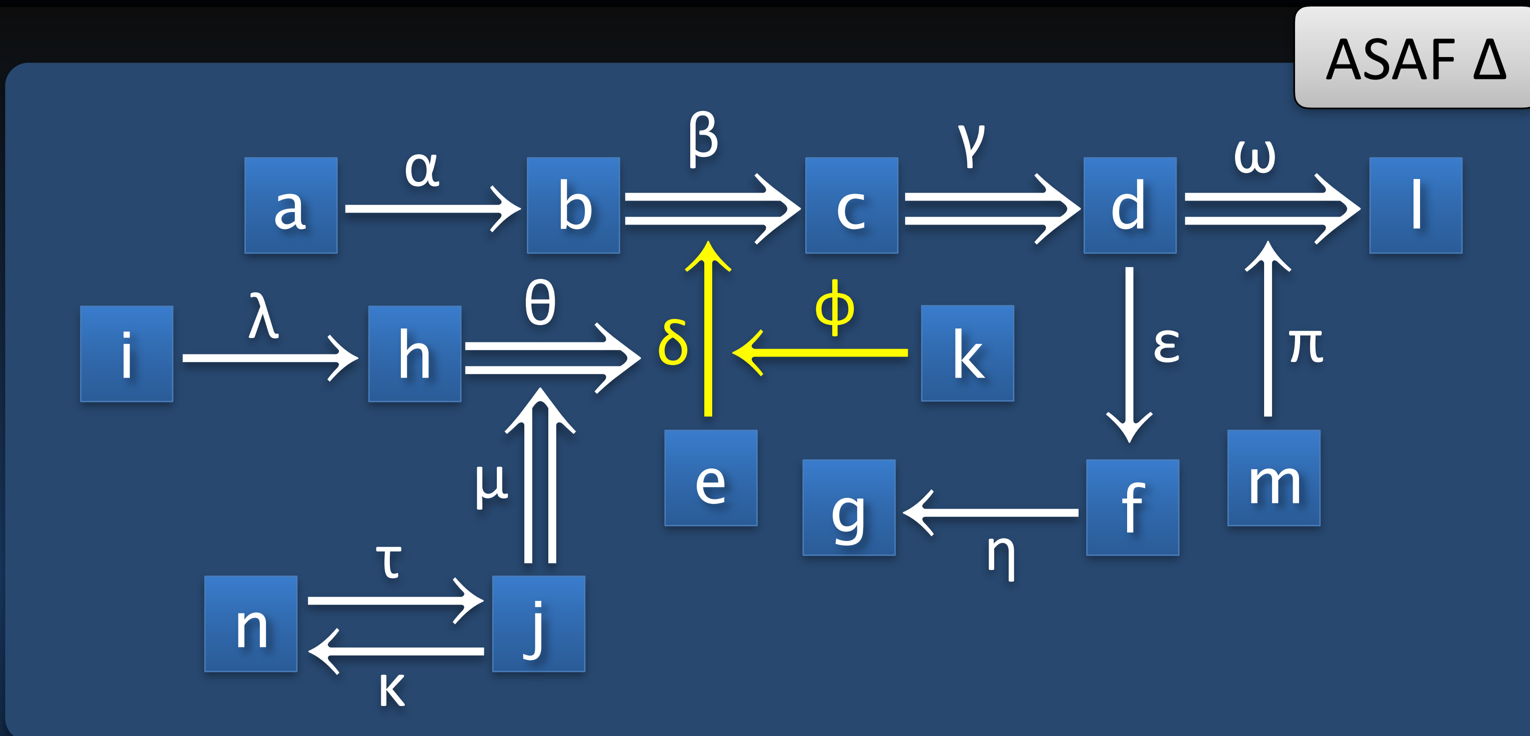

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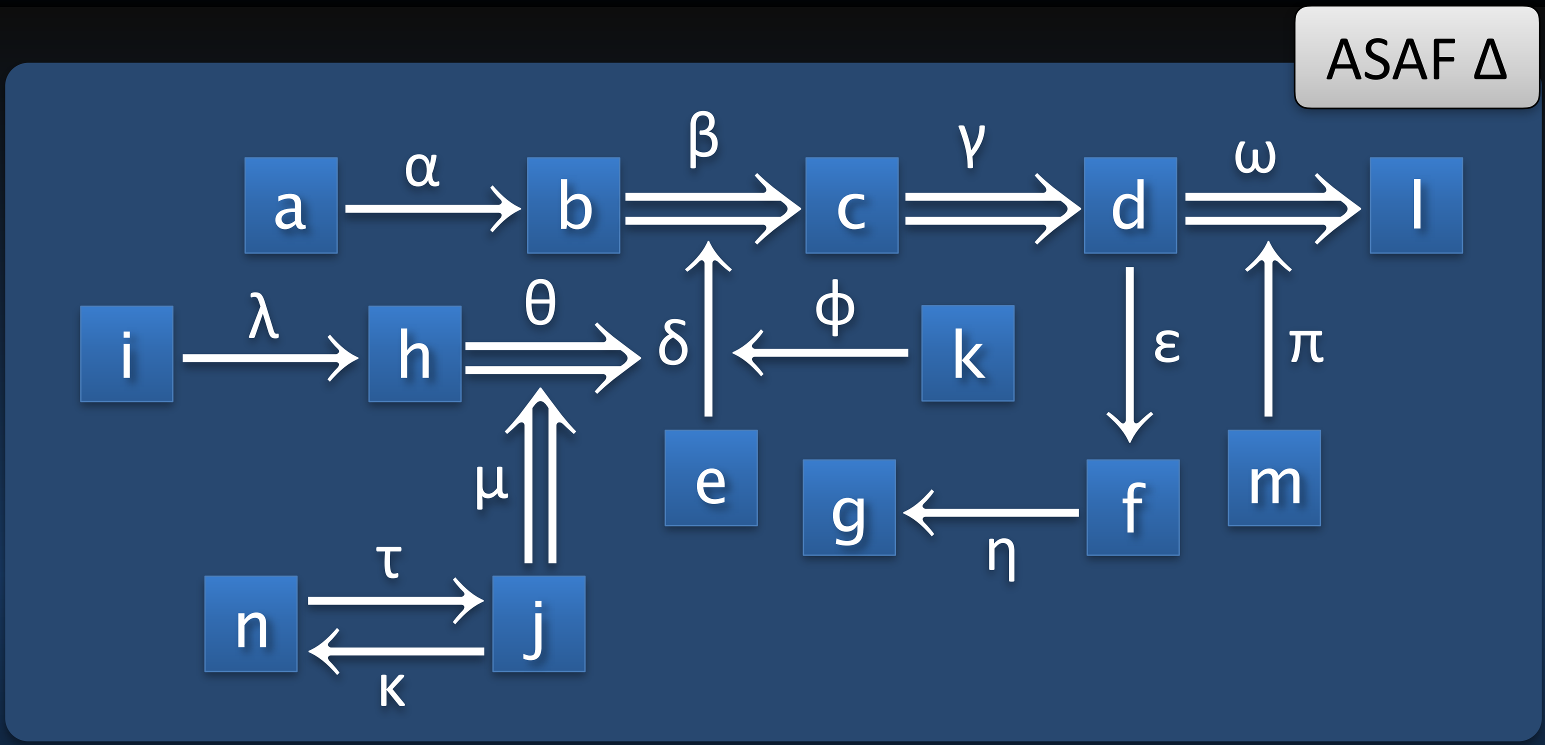

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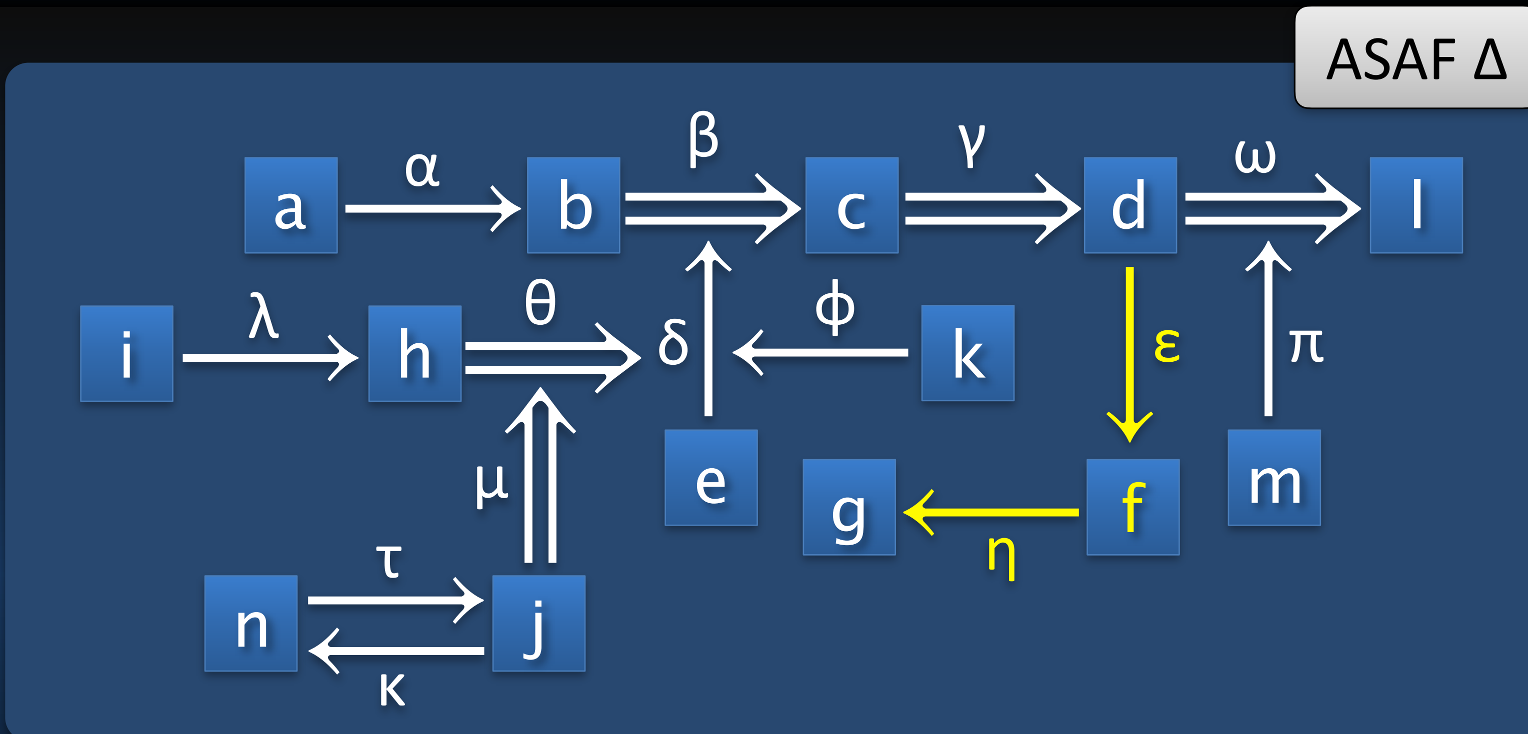

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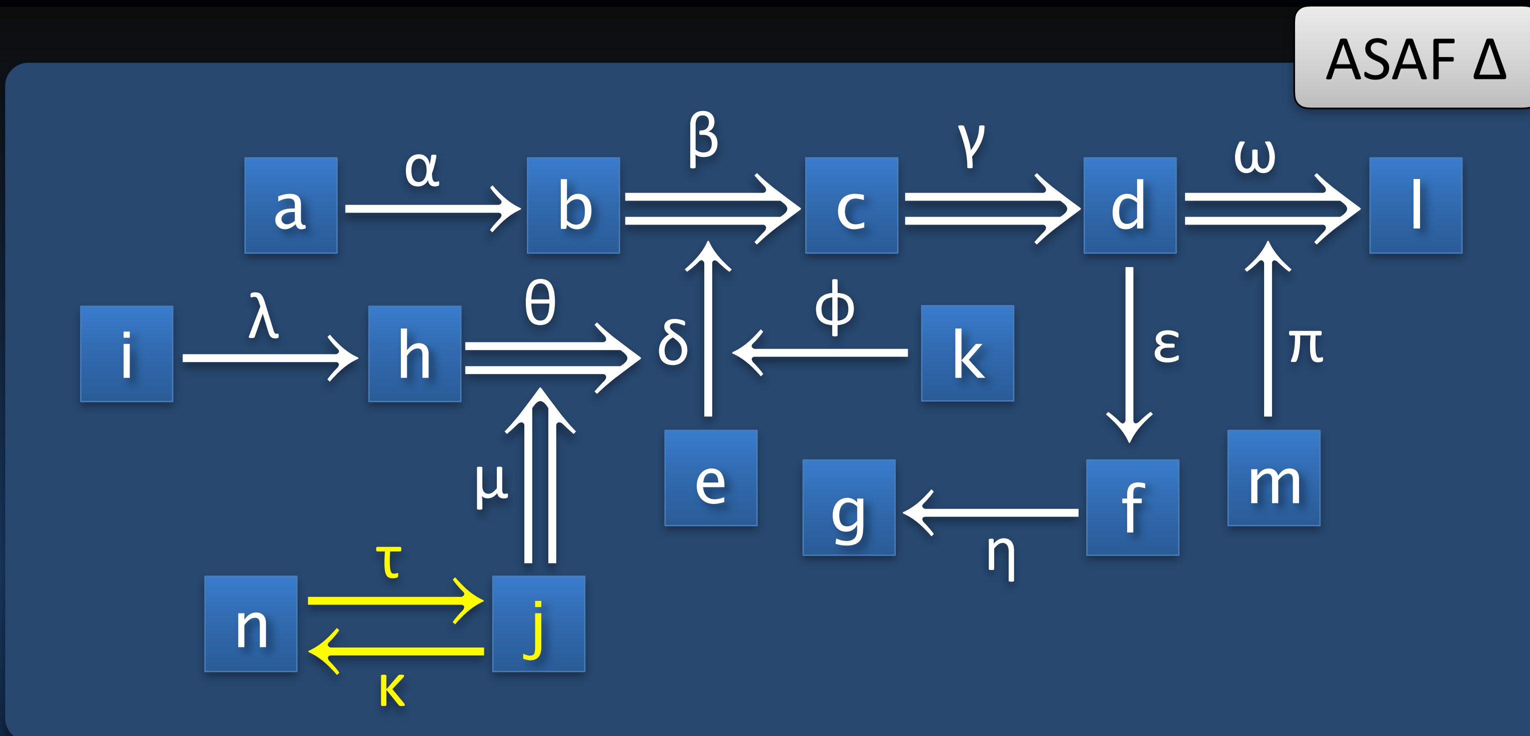

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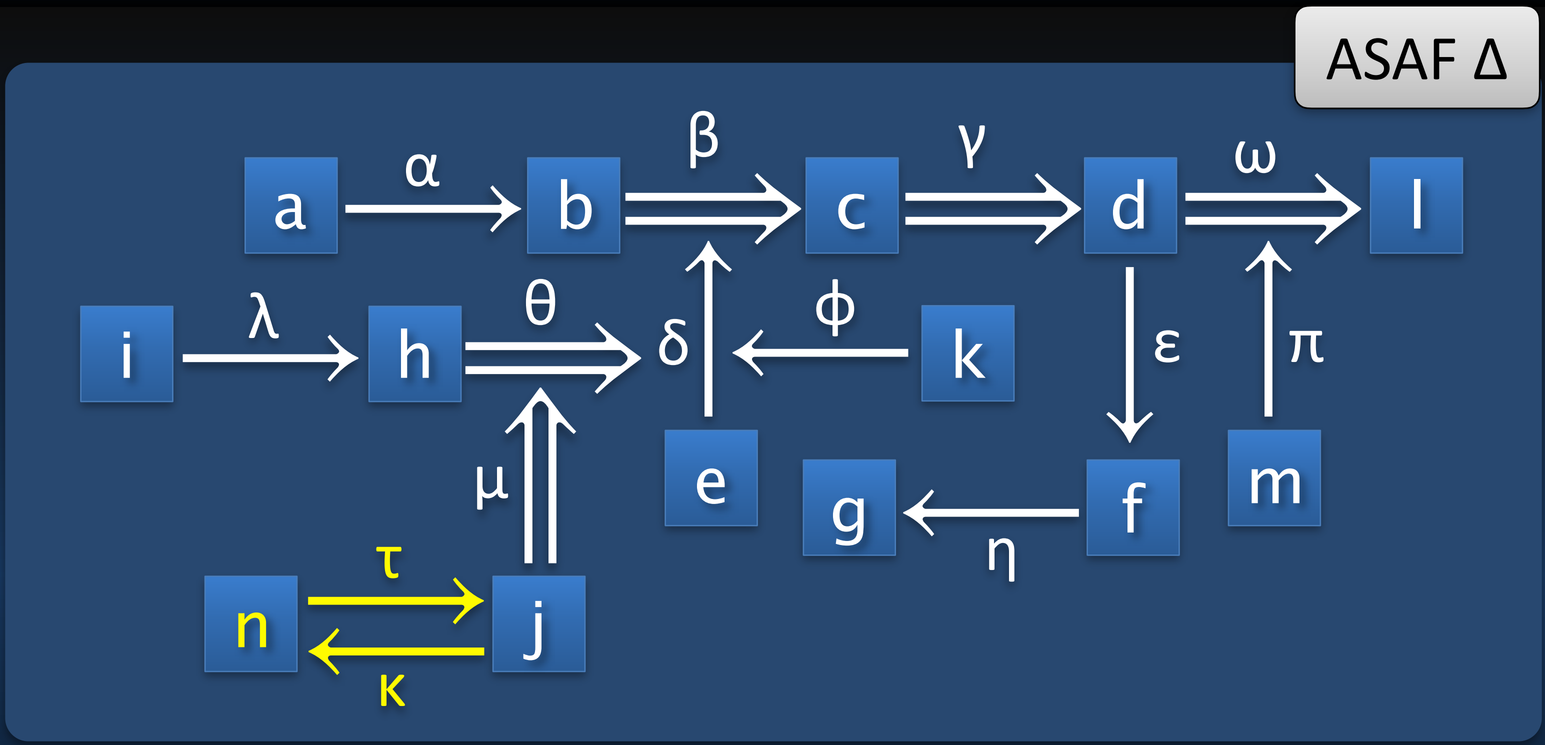

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- Conditional defeats are meant to enforce the acceptability constraints associated with the necessary support relation.

- Let $\langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $X \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$:
  - $\Sigma = [A_1,\ldots,A_n]$ is a support sequence for X ($n \geq 2$) iff $A_n = X$ and $\forall A_i$ ($1 \leq i \leq n-1$): $(A_i, A_{i+1}) \in \mathbb{S}$.
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  - Extended-Indirect Defeat: Let $\alpha, \beta \in \mathbb{R}$ and $\mathbf{S} \subseteq \mathbb{S}$.
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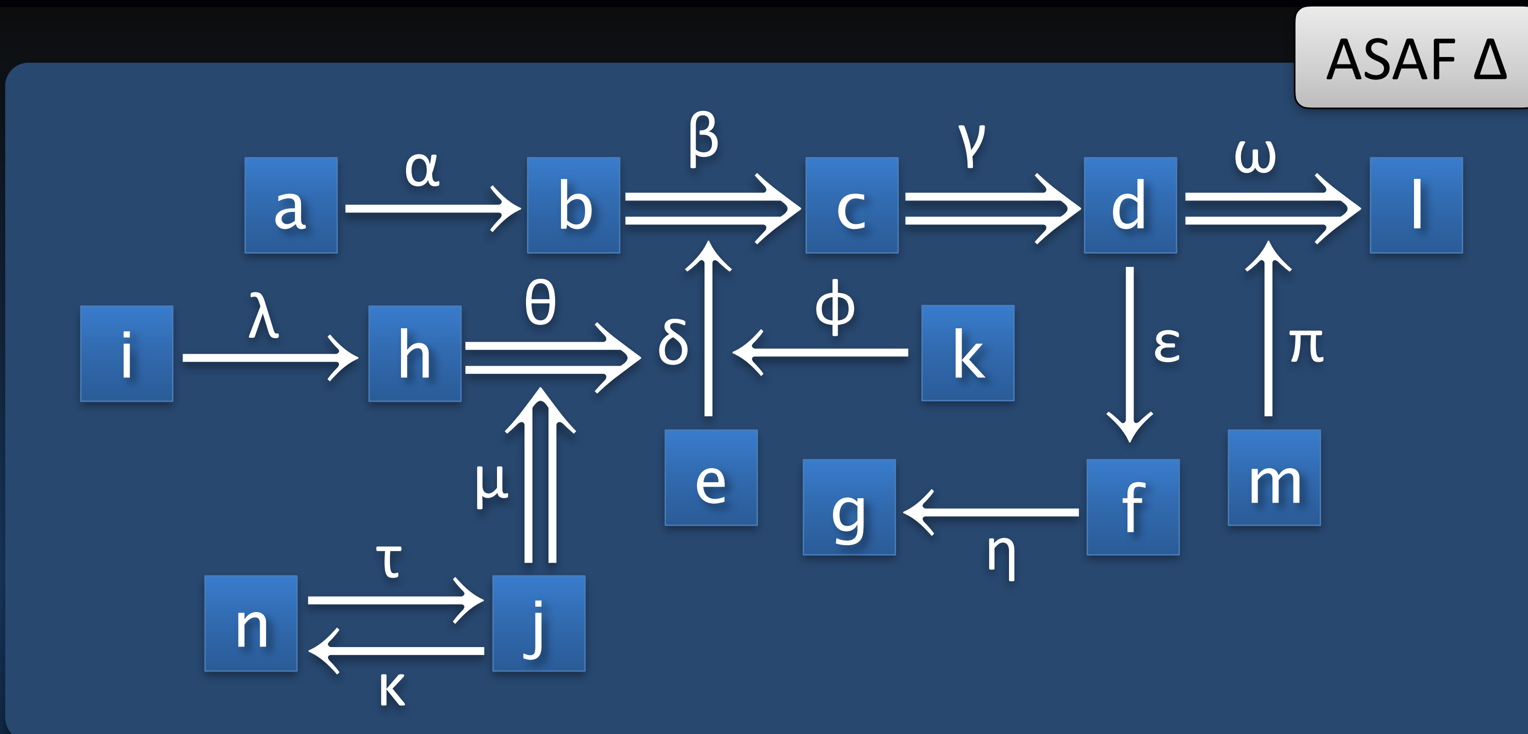

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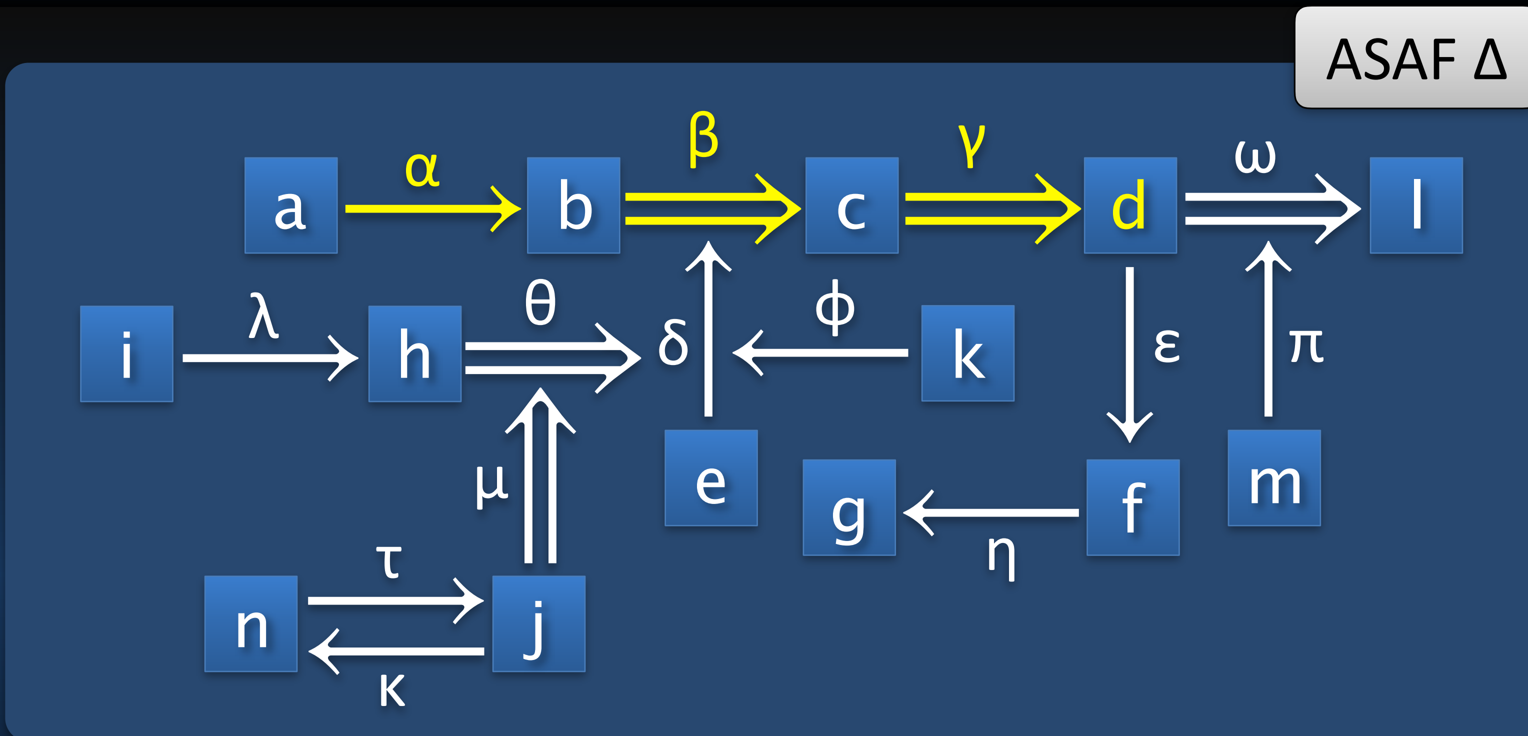

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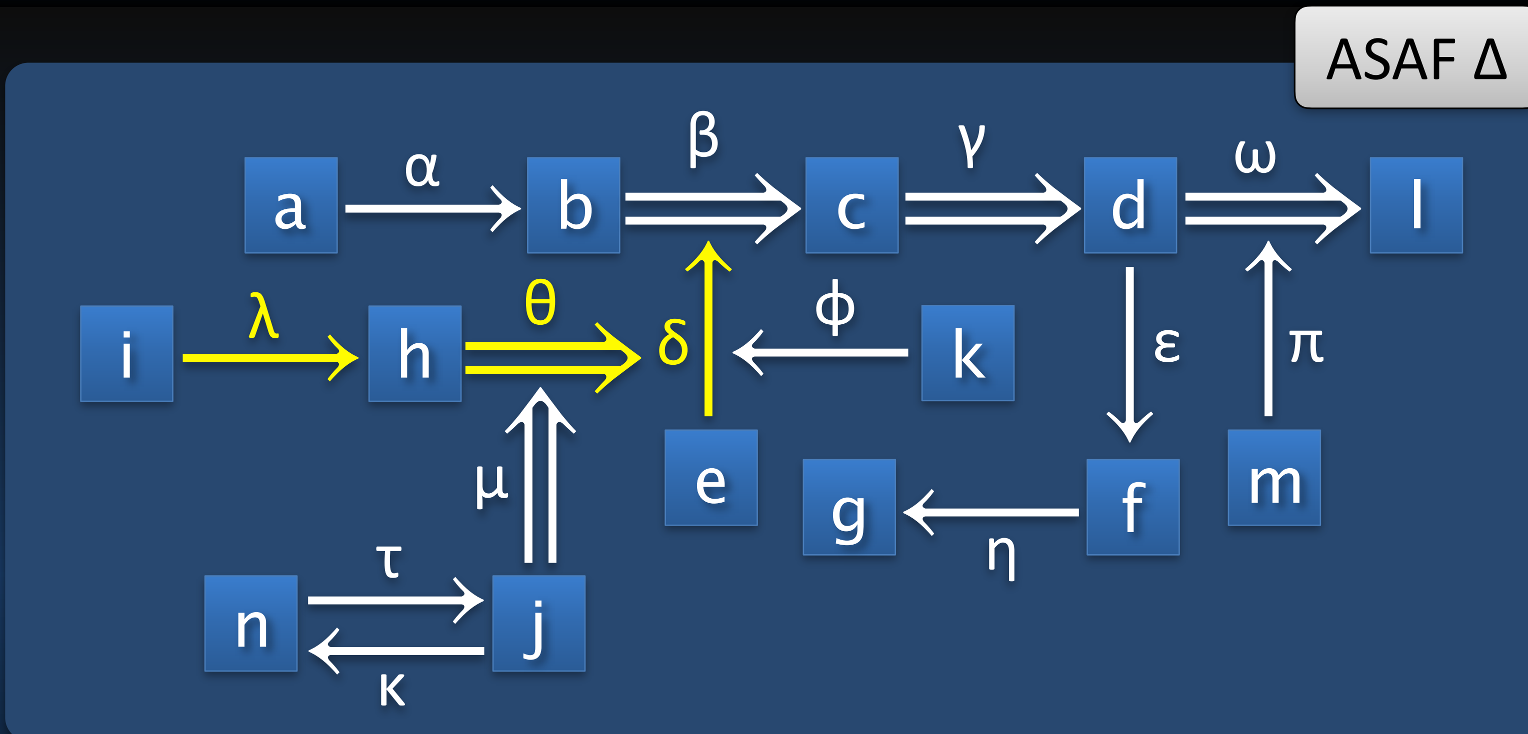

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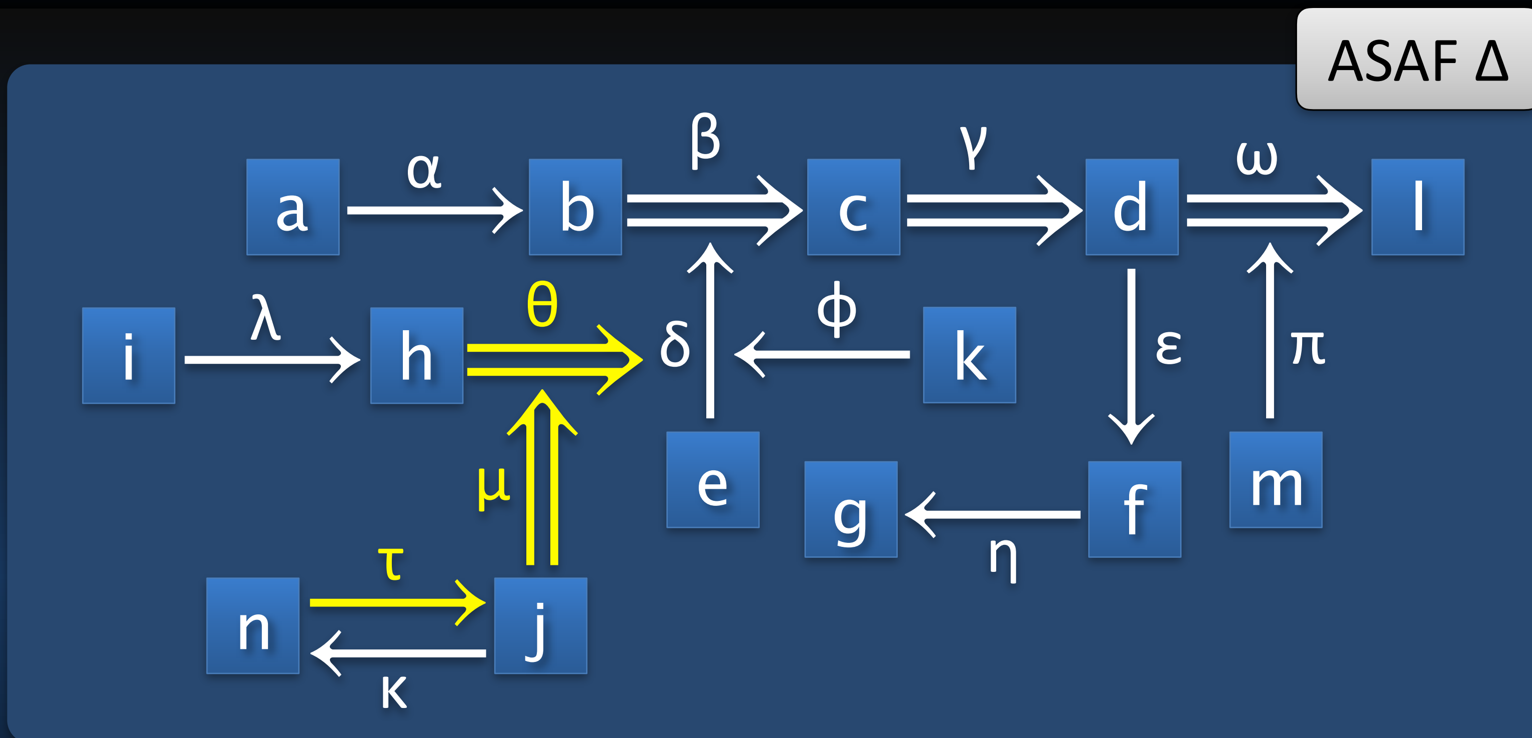

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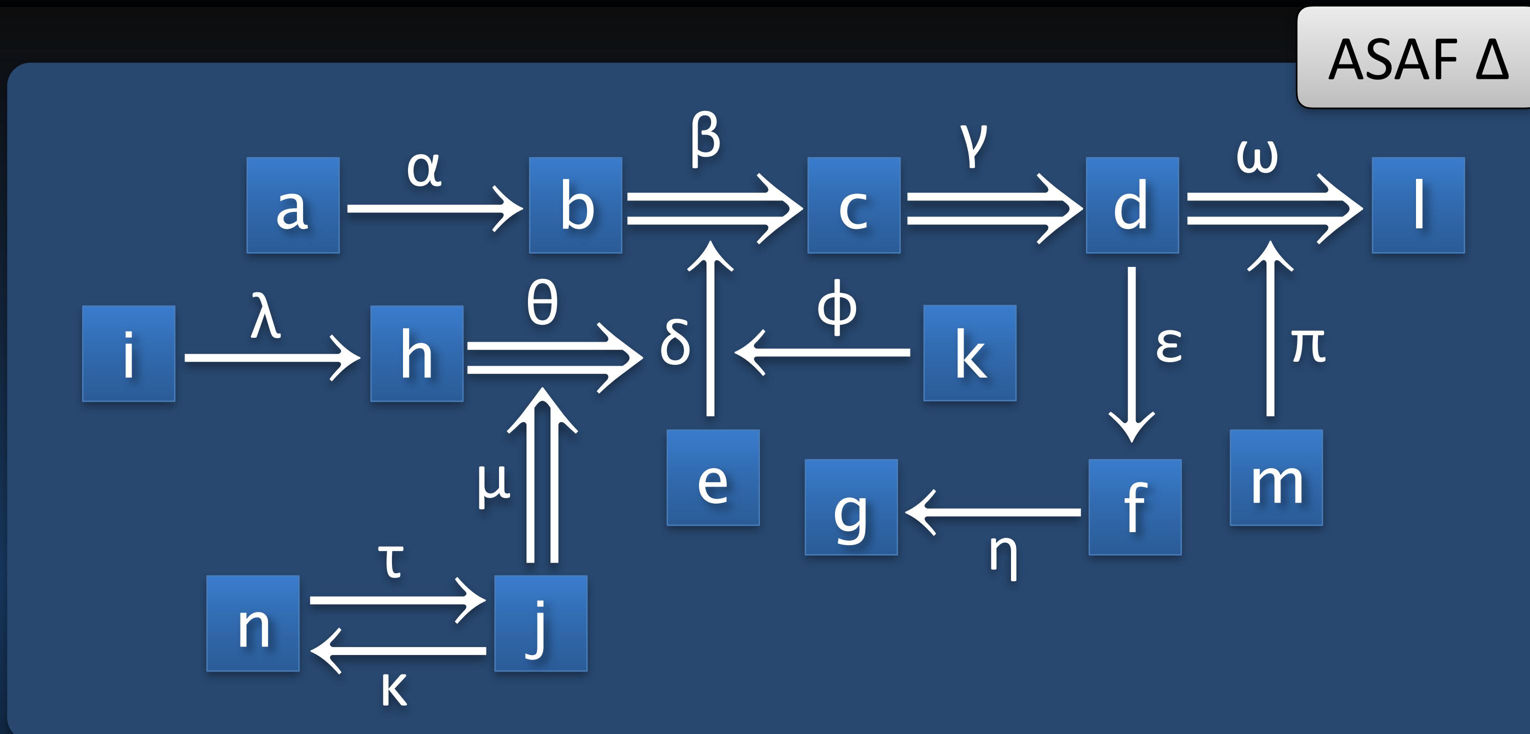

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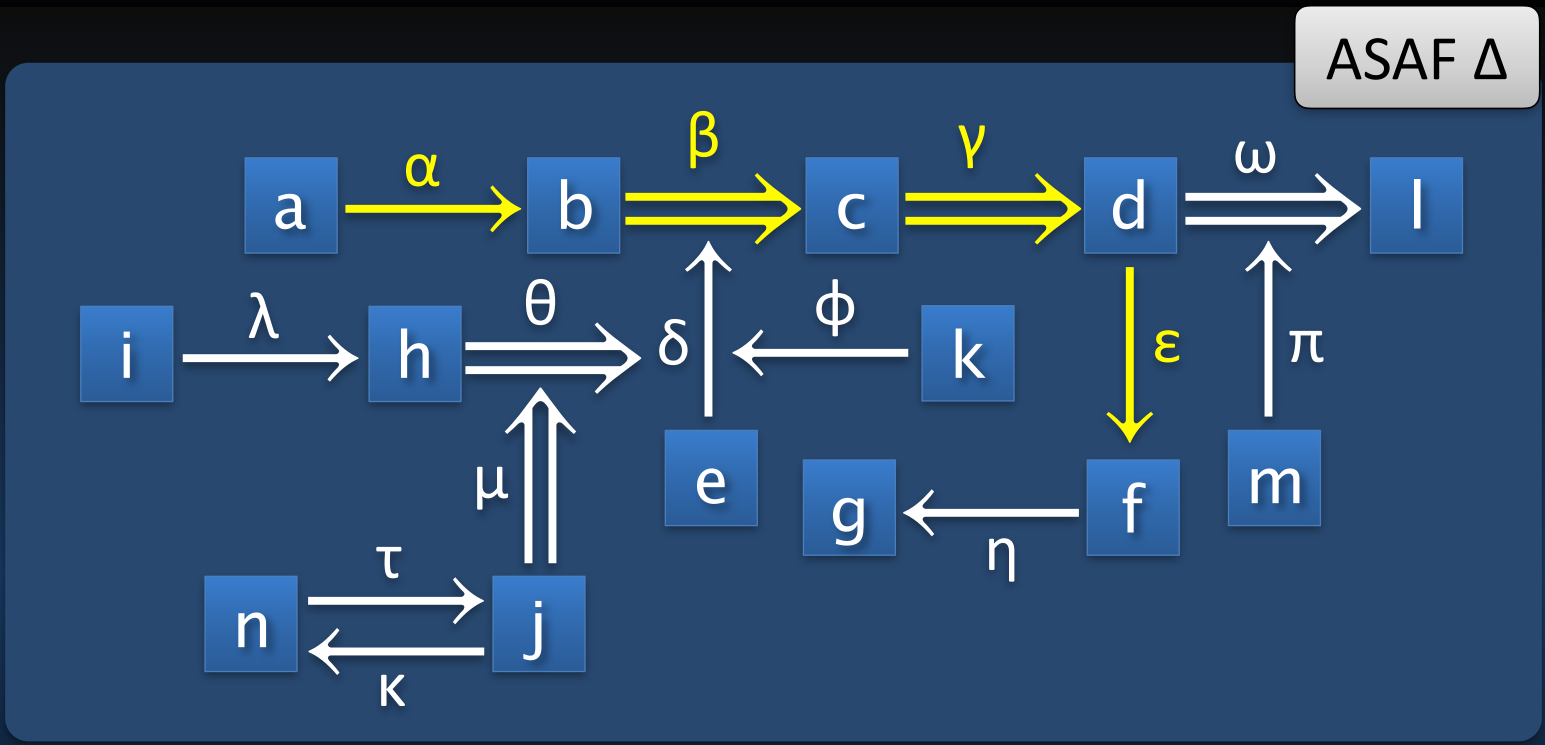

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Basic Semantic Notions of the ASAF

ASAF - Conflict-freeness

- We adapt Dung's notion of conflict-freeness in order to account for unconditional and conditional defeats in the ASAF.

- Let $\langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $\mathbf{S} \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$. We say that S is conflict-free if:
  - $\nexists \alpha, X \in \mathbf{S}$ s.t. α u-def X; and
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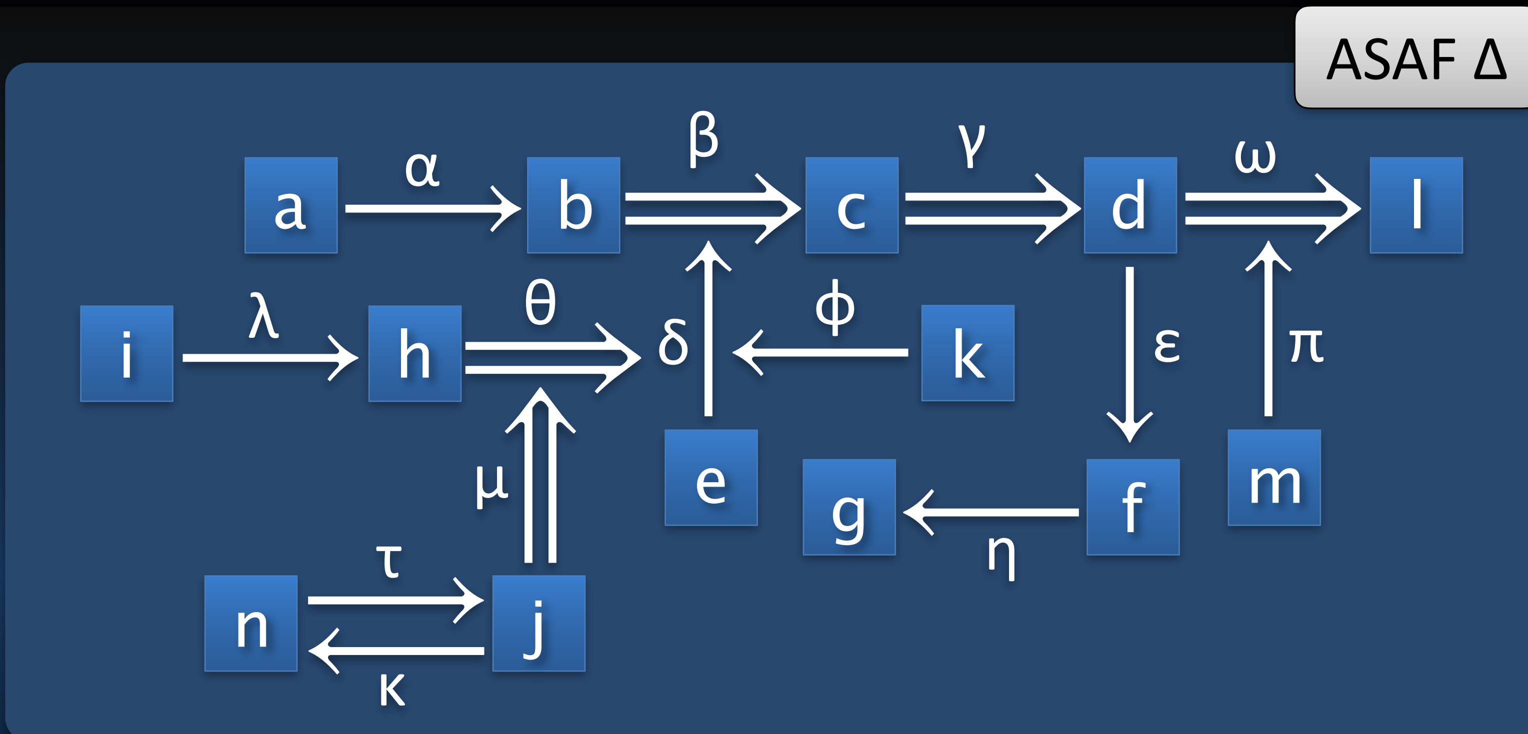

- Conflict-free Sets: {a,b,c,d,e,f,g,h,i,j,k,l,m,n,β,γ,ω,θ,μ}, {λ,δ} and {α,β,ε}.

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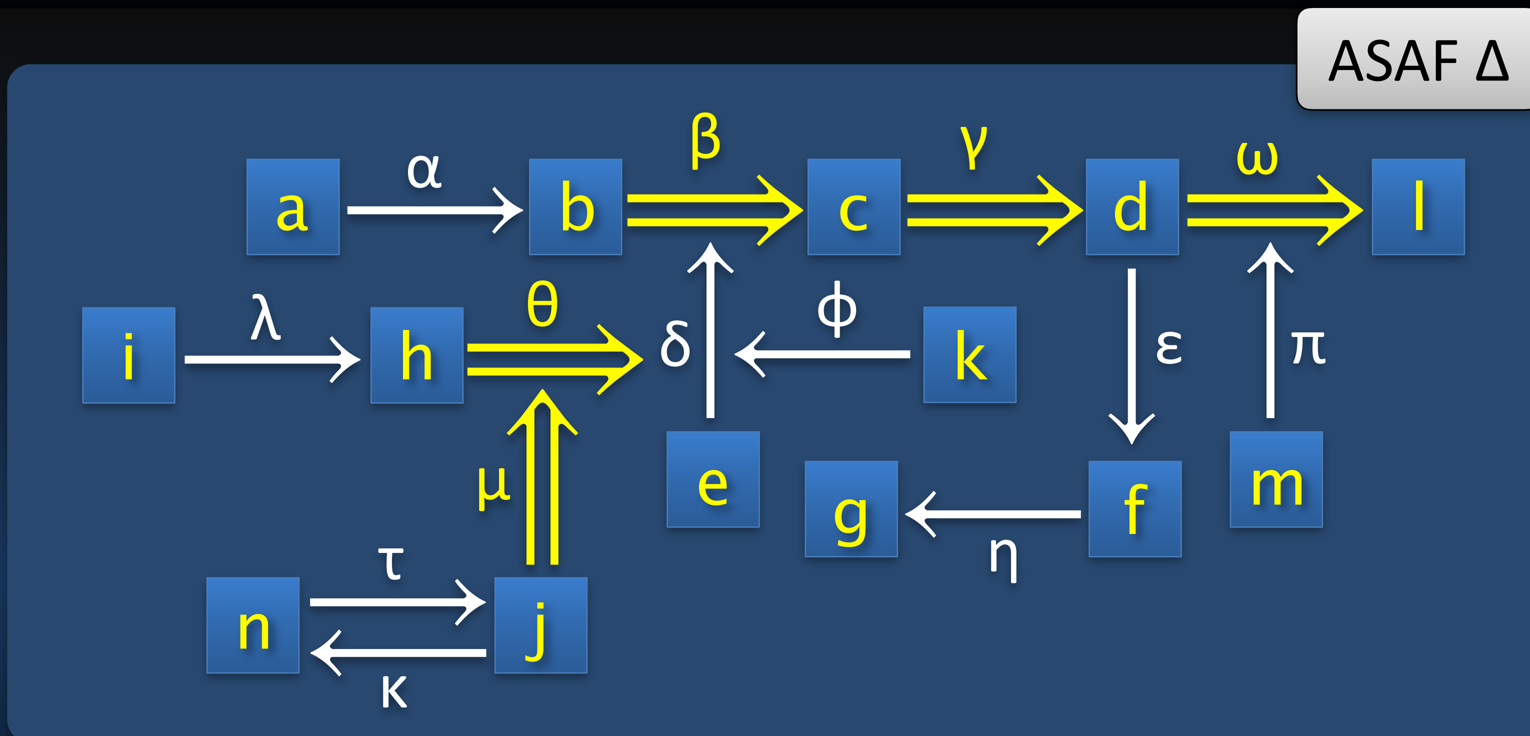

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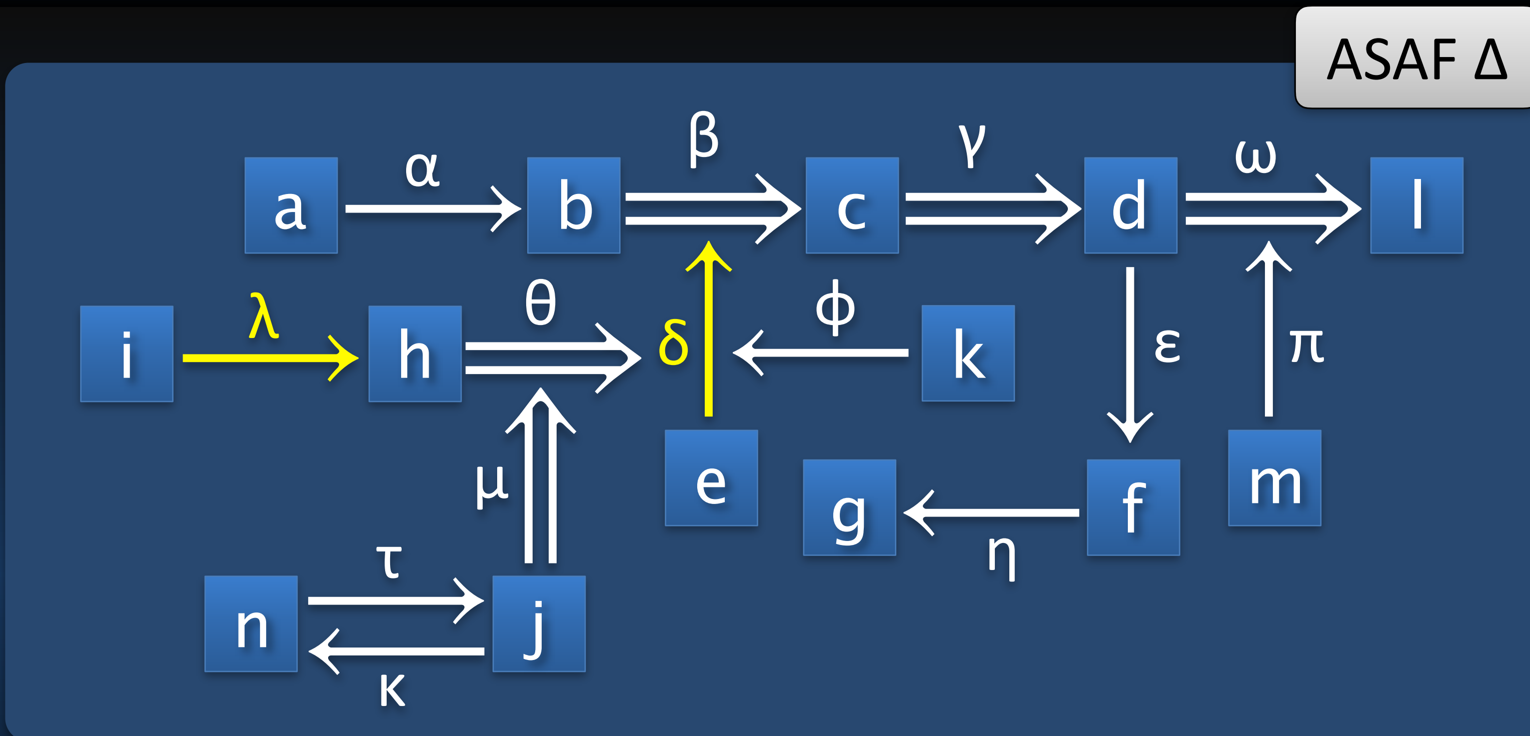

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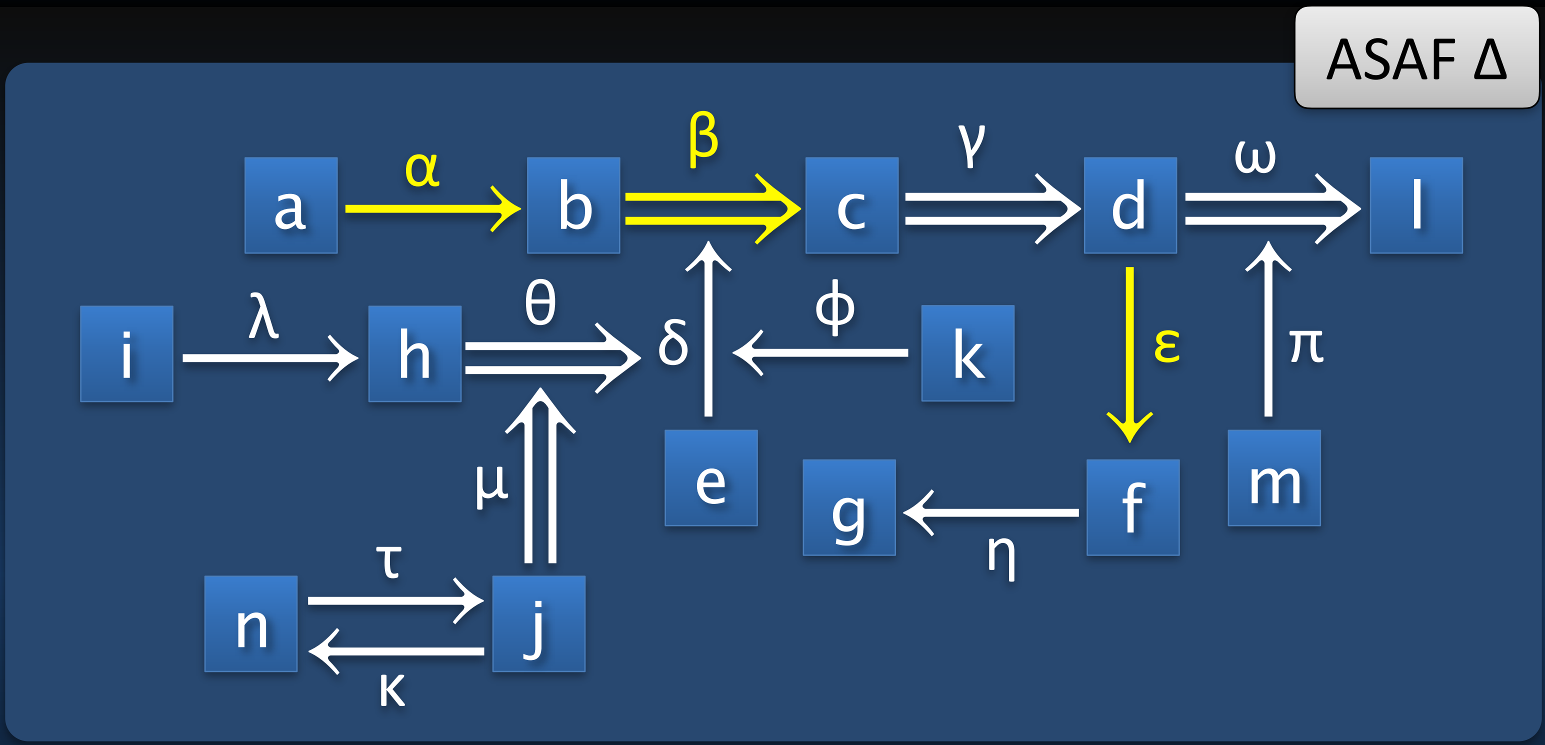

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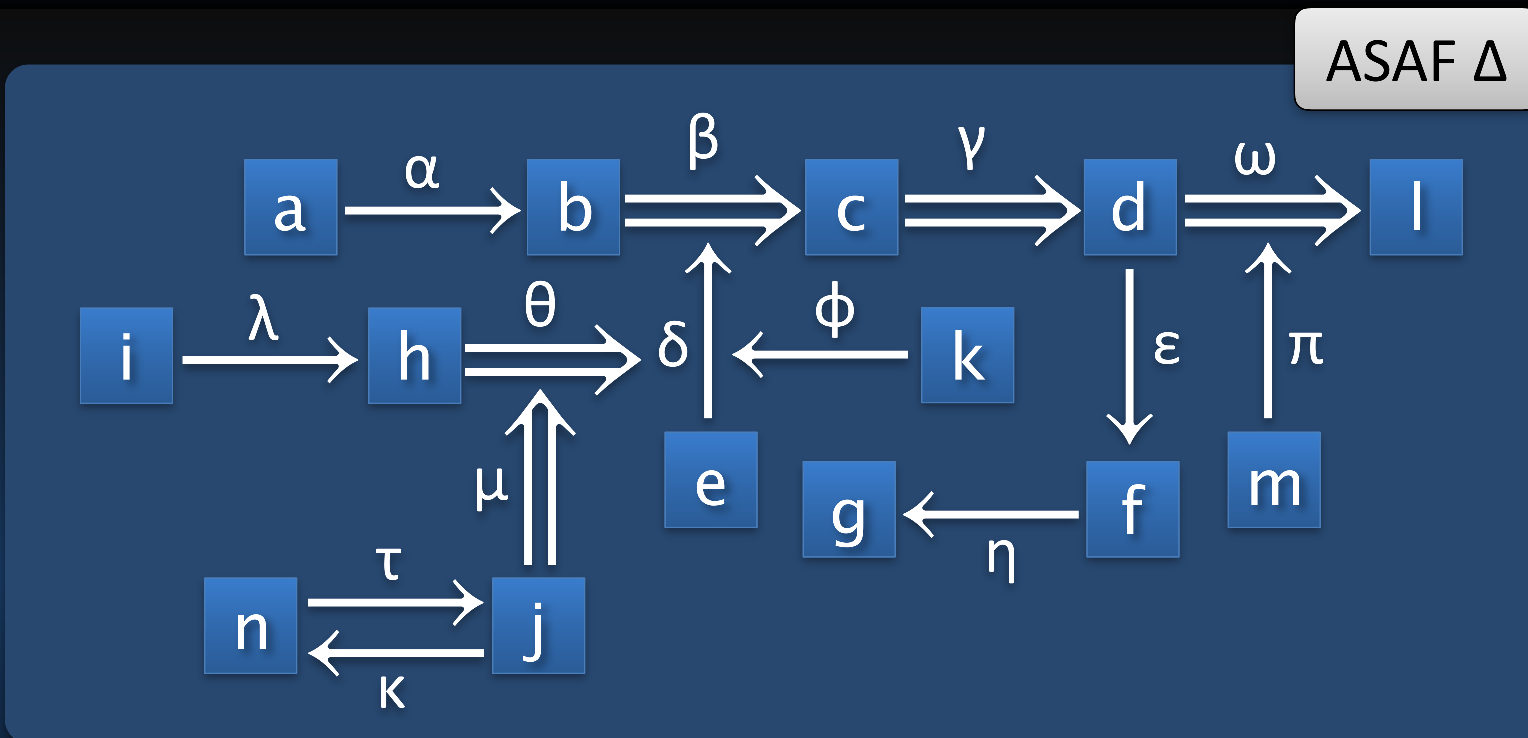
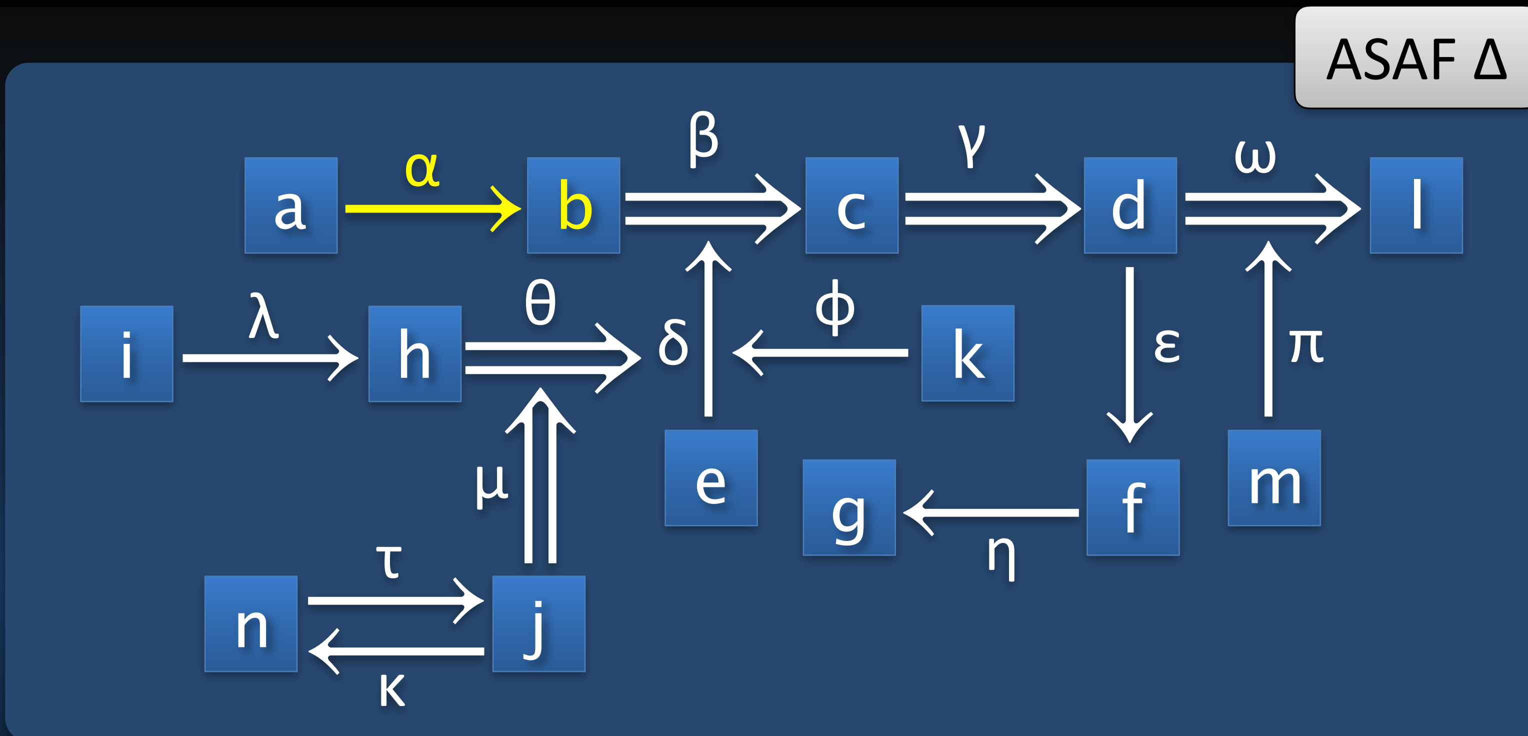


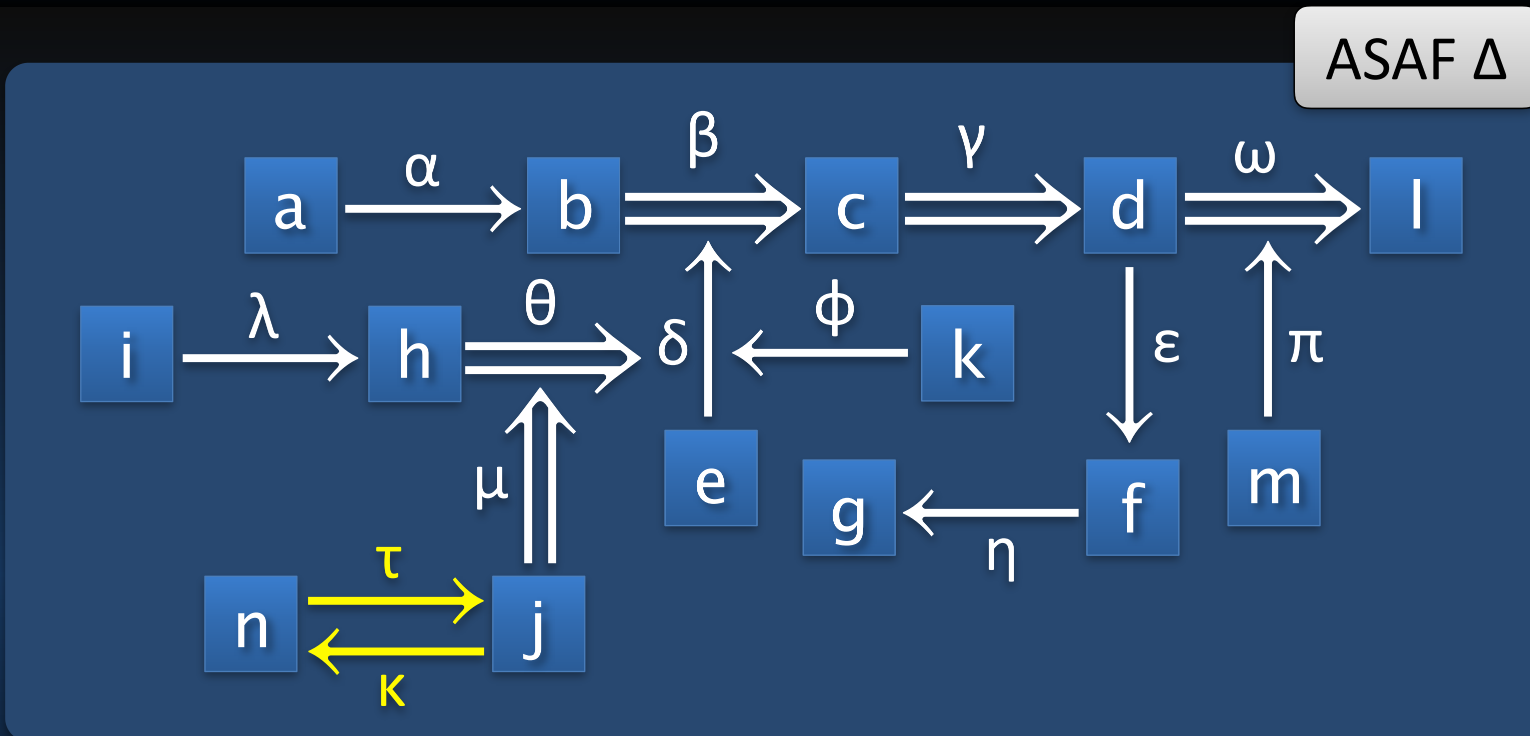
- Non-conflict-free Sets: {α,b}, {τ,κ}, {λ,θ,δ} and {α,β,γ,ε}.

ASAF - Conflict-freeness Example



- Non-conflict-free Sets: {α,b}, {τ,κ}, {λ,θ,δ} and {α,β,γ,ε}.

ASAF - Conflict-freeness Example



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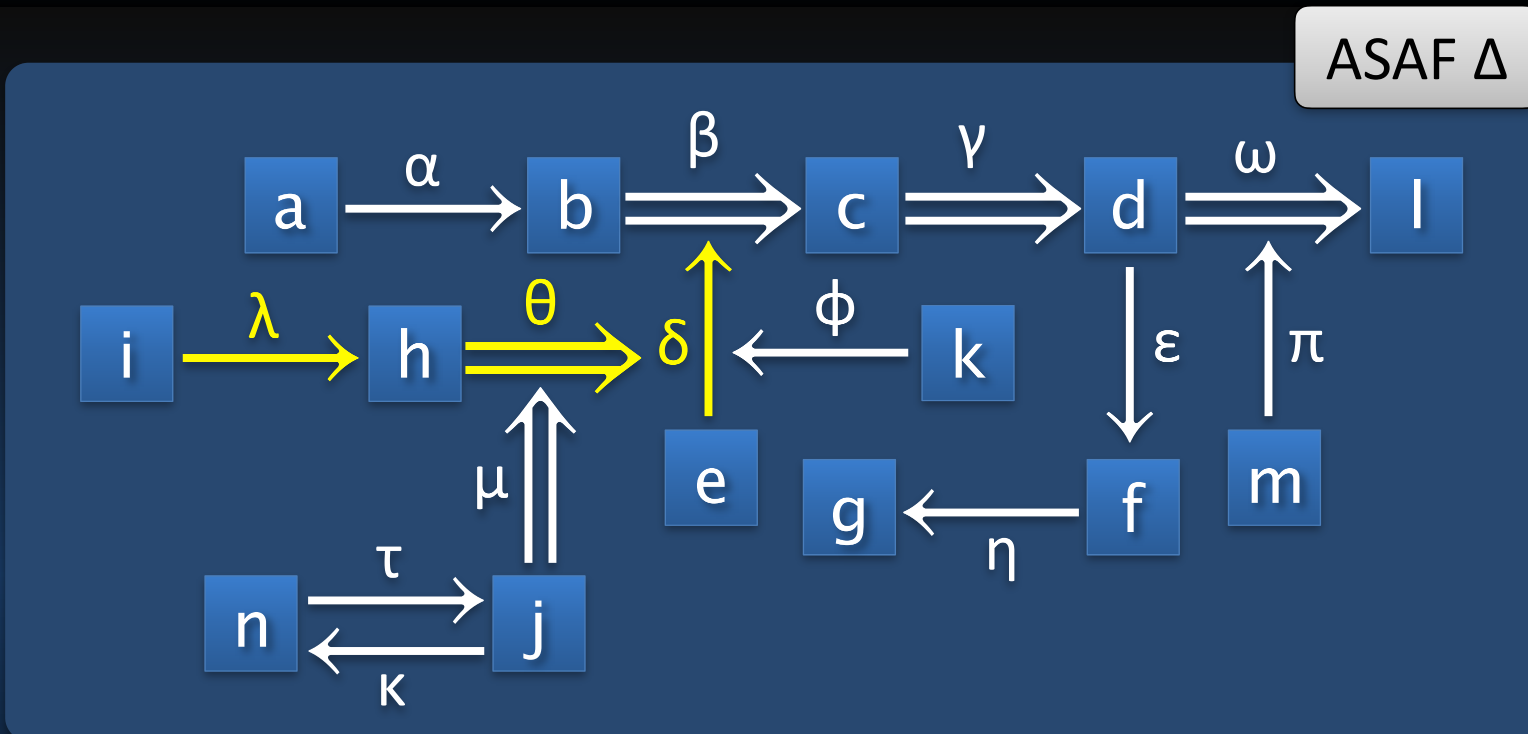

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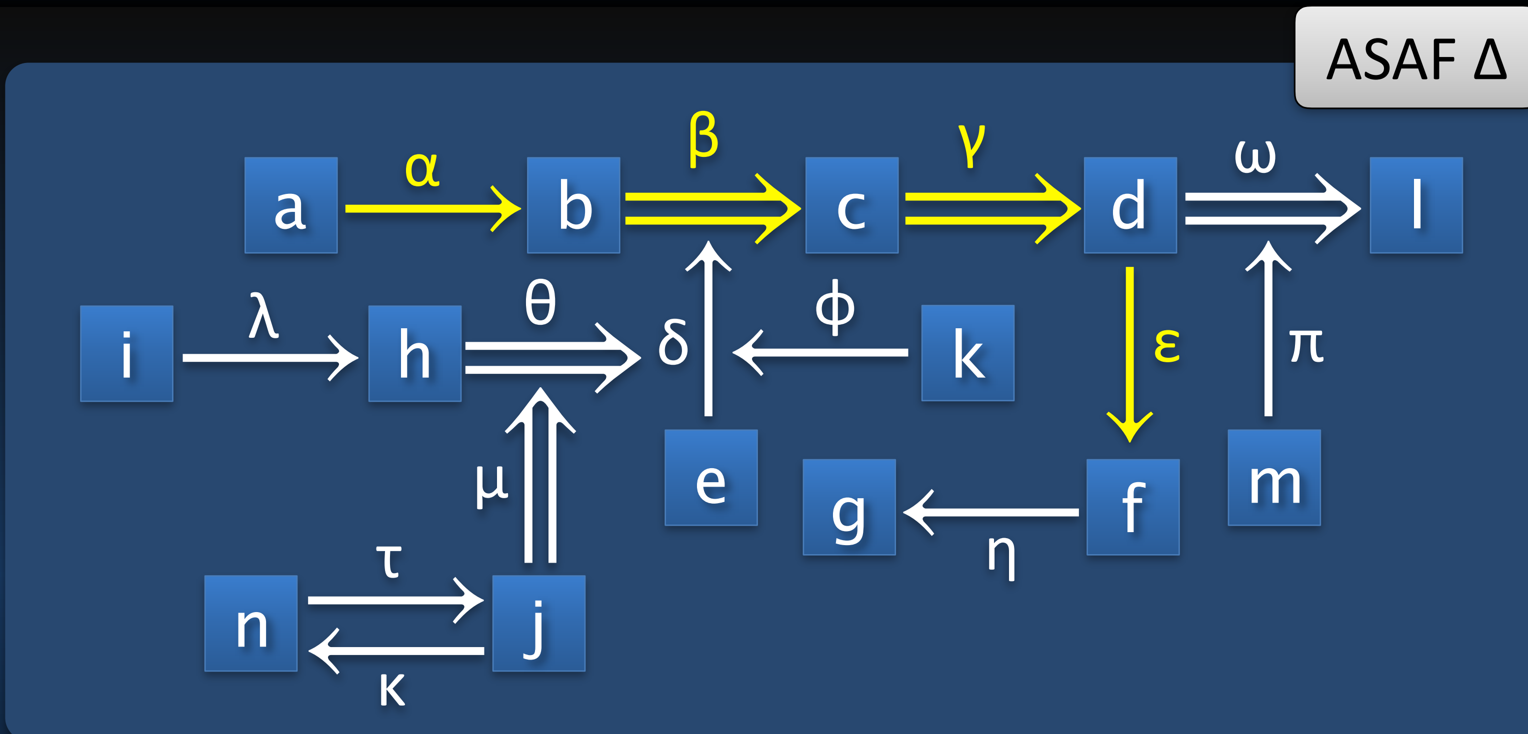

- Non-conflict-free Sets: {α,b}, {τ,κ}, {λ,θ,δ} and {α,β,γ,ε}.

ASAF - Acceptability

- The notion of acceptability characterizes the defense by a set of arguments, attacks and supports of the ASAF against unconditional and conditional defeats on its elements.

- Let $\langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF, $X \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ and $\mathbf{S} \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$. We say that X is acceptable w.r.t. $\mathbf{S}$ if:

1. $\forall \alpha \in \mathbb{R}$ s.t. α u-def X, either:
   (a) $\exists \beta \in \mathbf{S}$ s.t. β u-def α; or
   (b) $\exists \beta \in \mathbf{S}, \exists \mathbf{S}' \subseteq \mathbf{S}$ s.t. β c-def α given $\mathbf{S}'$.
2. $\forall \alpha \in \mathbb{R}, \forall \mathbf{T} \subseteq \mathbb{S}$ s.t. α c-def X given $\mathbf{T}$, either:
   (a) $\exists \beta \in \mathbf{S}$ s.t. β u-def α;
   (b) $\exists \beta \in \mathbf{S}, \exists \gamma \in \mathbf{T}$ s.t. β u-def γ;
   (c) $\exists \beta \in \mathbf{S}, \exists \mathbf{S}' \subseteq \mathbf{S}$ s.t. β c-def α given $\mathbf{S}'$; or
   (d) $\exists \beta \in \mathbf{S}, \exists \mathbf{S}' \subseteq \mathbf{S}, \exists \gamma \in \mathbf{T}$ s.t. β c-def γ given $\mathbf{S}'$.

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1. $\forall \alpha \in \mathbb{R}$ s.t. α u-def X, either:
   (a) $\exists \beta \in \mathbf{S}$ s.t. β u-def α; or
   (b) $\exists \beta \in \mathbf{S}, \exists \mathbf{S}' \subseteq \mathbf{S}$ s.t. β c-def α given $\mathbf{S}'$.
2. $\forall \alpha \in \mathbb{R}, \forall \mathbf{T} \subseteq \mathbb{S}$ s.t. α c-def X given **T**, either:
   (a) $\exists \beta \in \mathbf{S}$ s.t. β u-def α;
   (b) $\exists \beta \in \mathbf{S}, \exists \gamma \in \mathbf{T}$ s.t. β u-def γ;
   (c) $\exists \beta \in \mathbf{S}, \exists \mathbf{S}' \subseteq \mathbf{S}$ s.t. β c-def α given $\mathbf{S}'$; or
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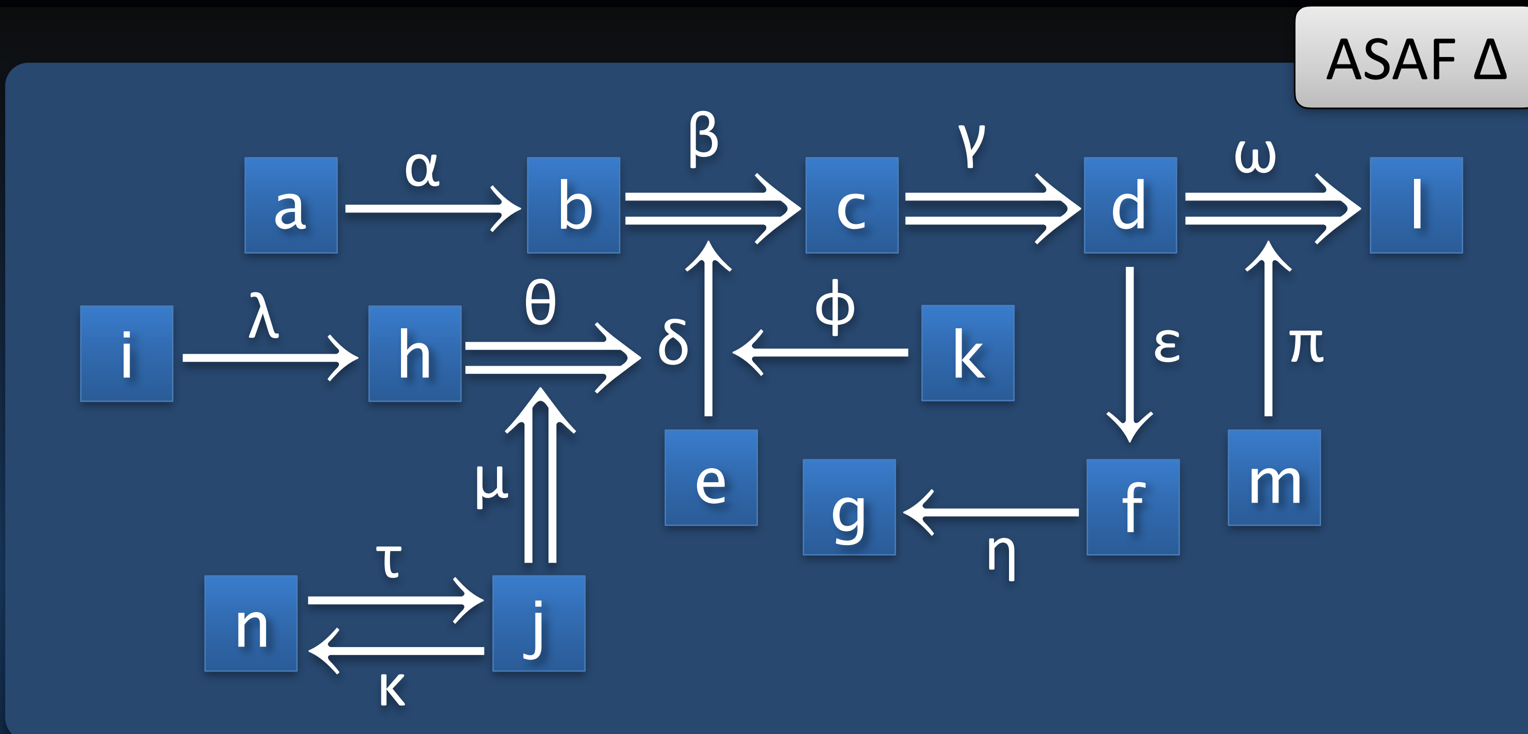
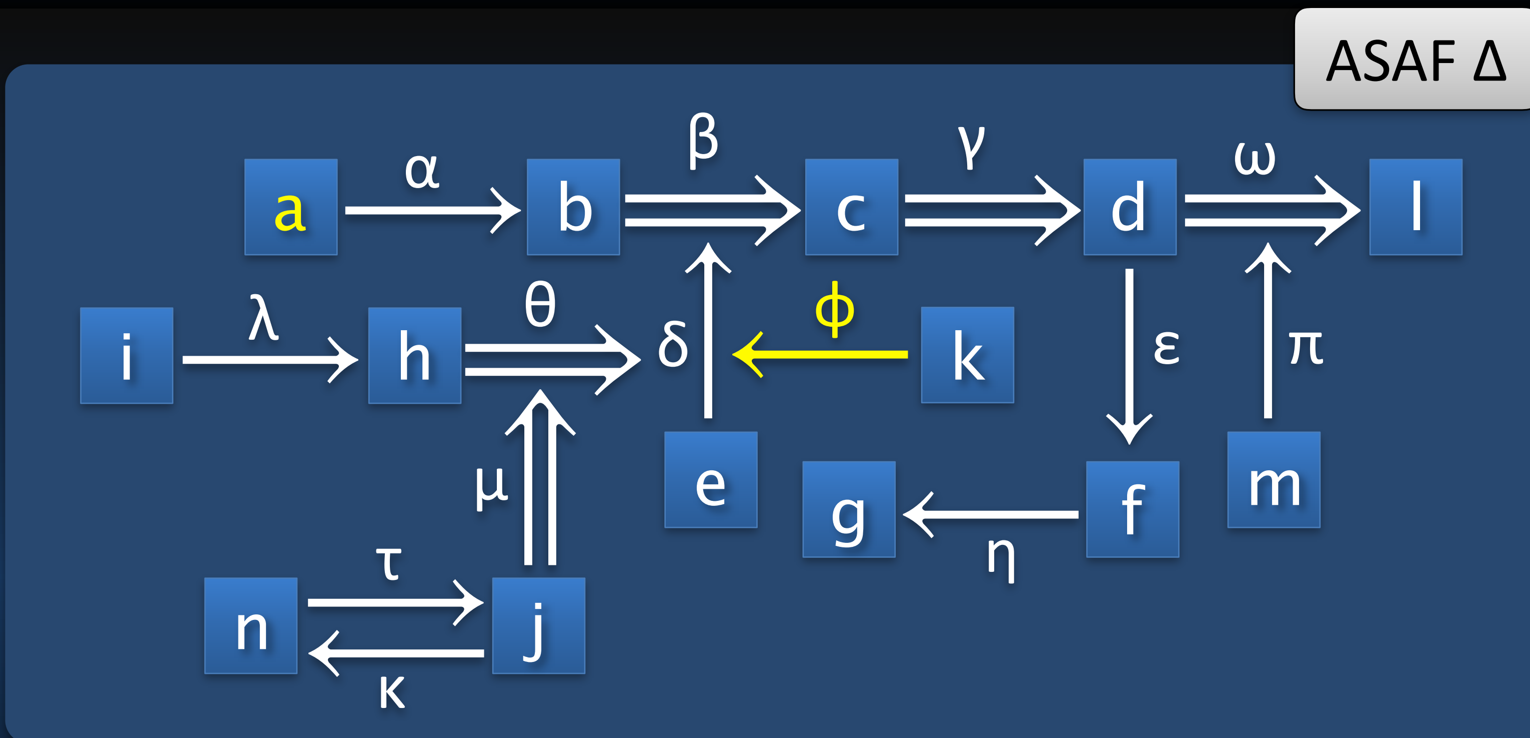


- a and ϕ are acceptable w.r.t. ∅,
- β is acceptable w.r.t. {ϕ},
- j and θ are acceptable w.r.t. {κ}; and
- η is acceptable w.r.t. {α,β,γ}

ASAF - Acceptability Example



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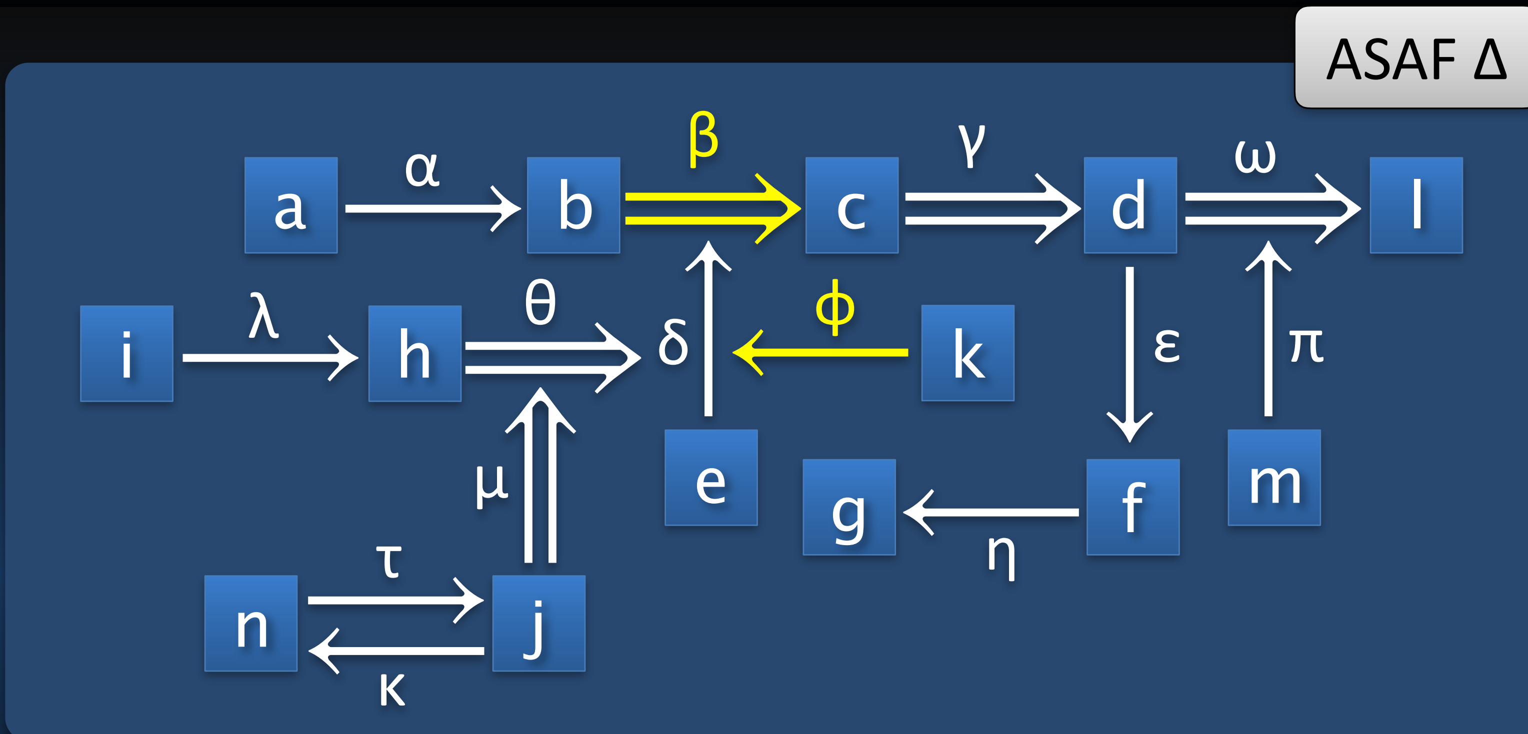

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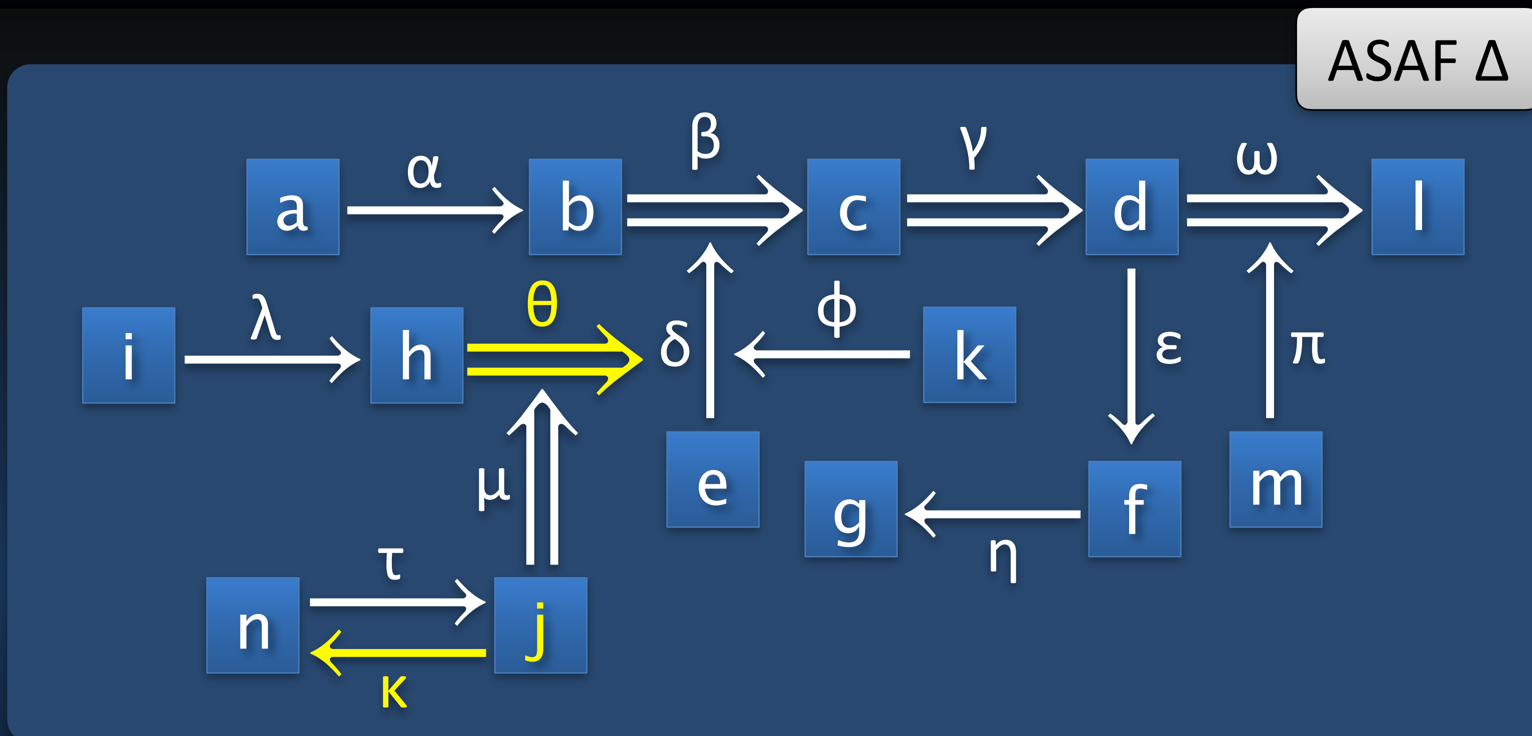

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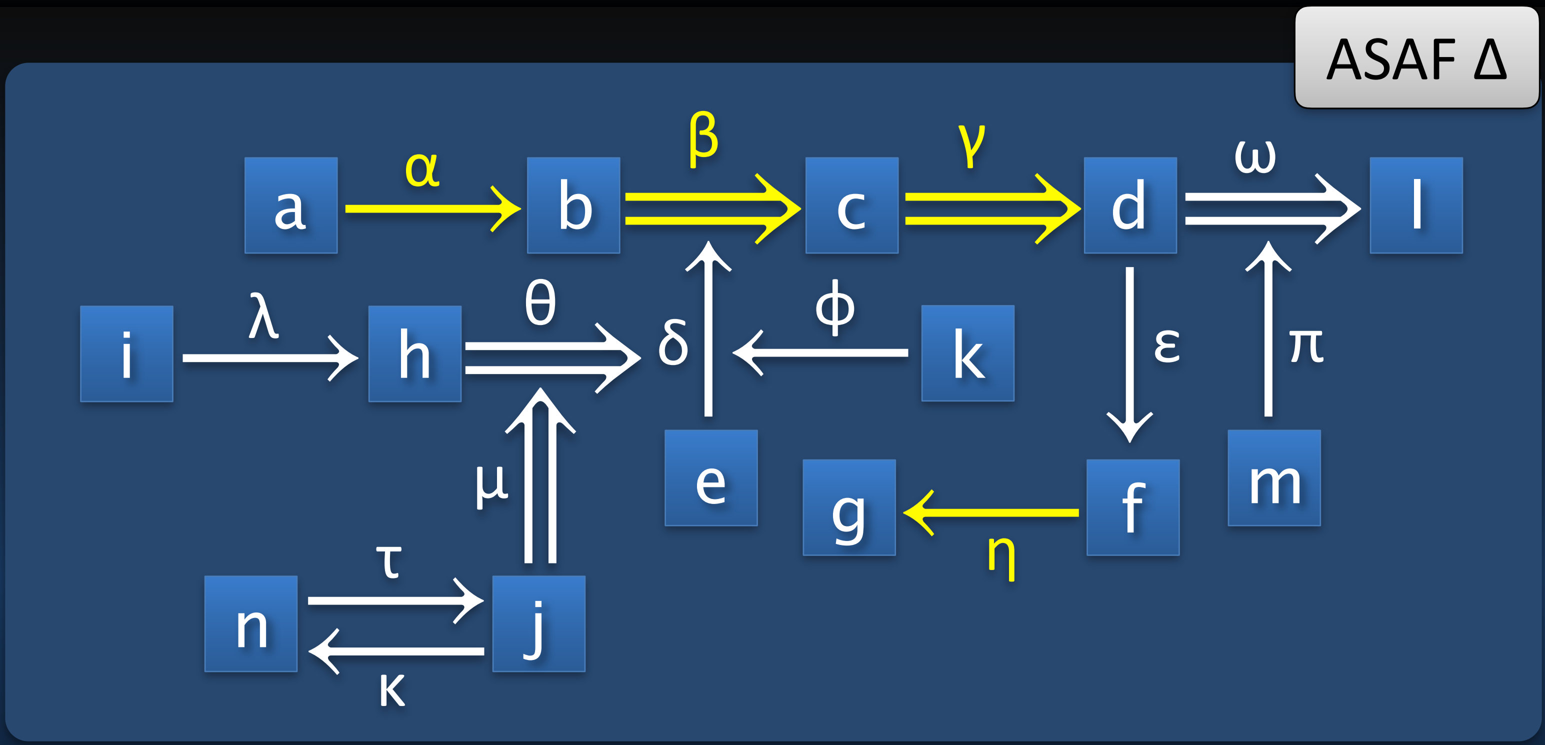

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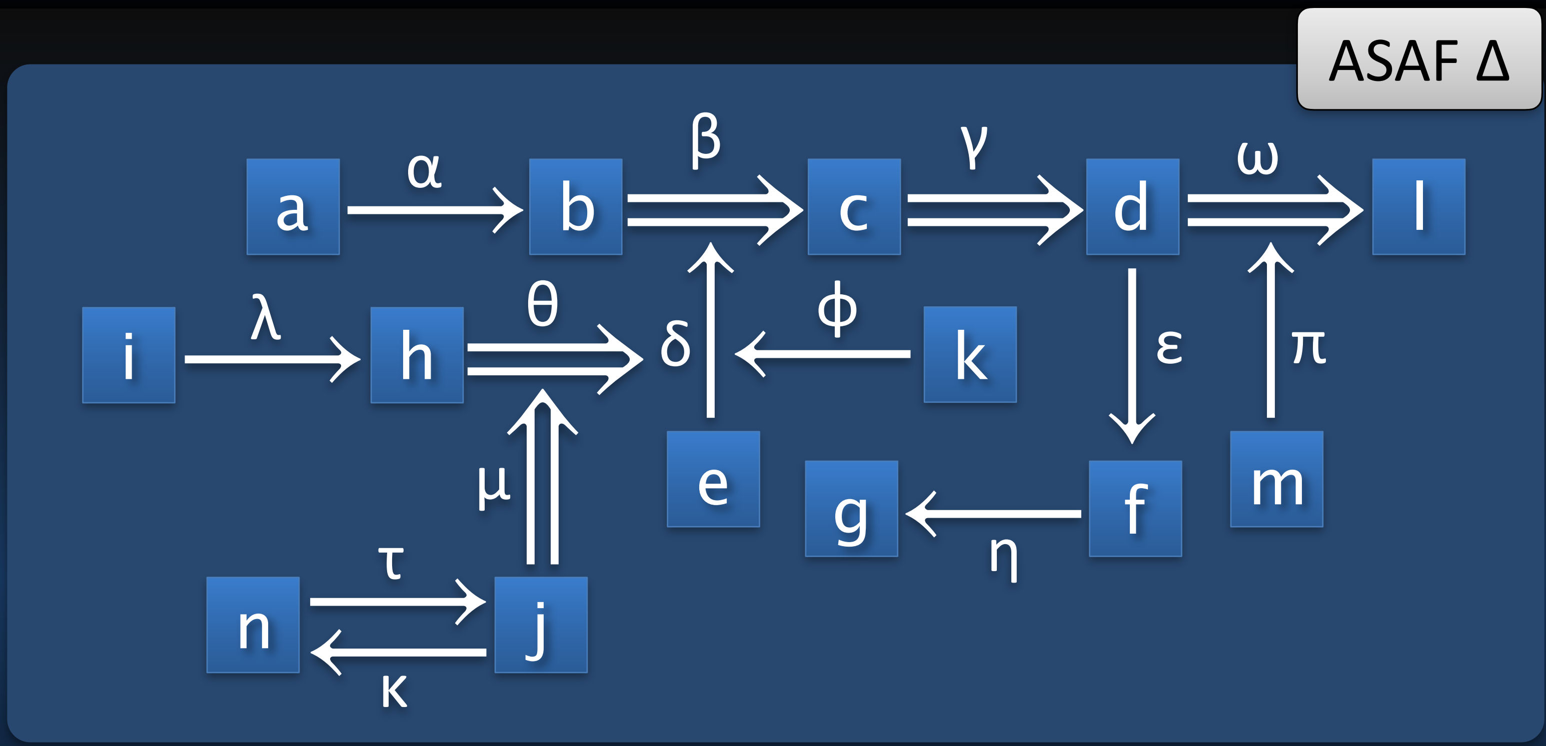
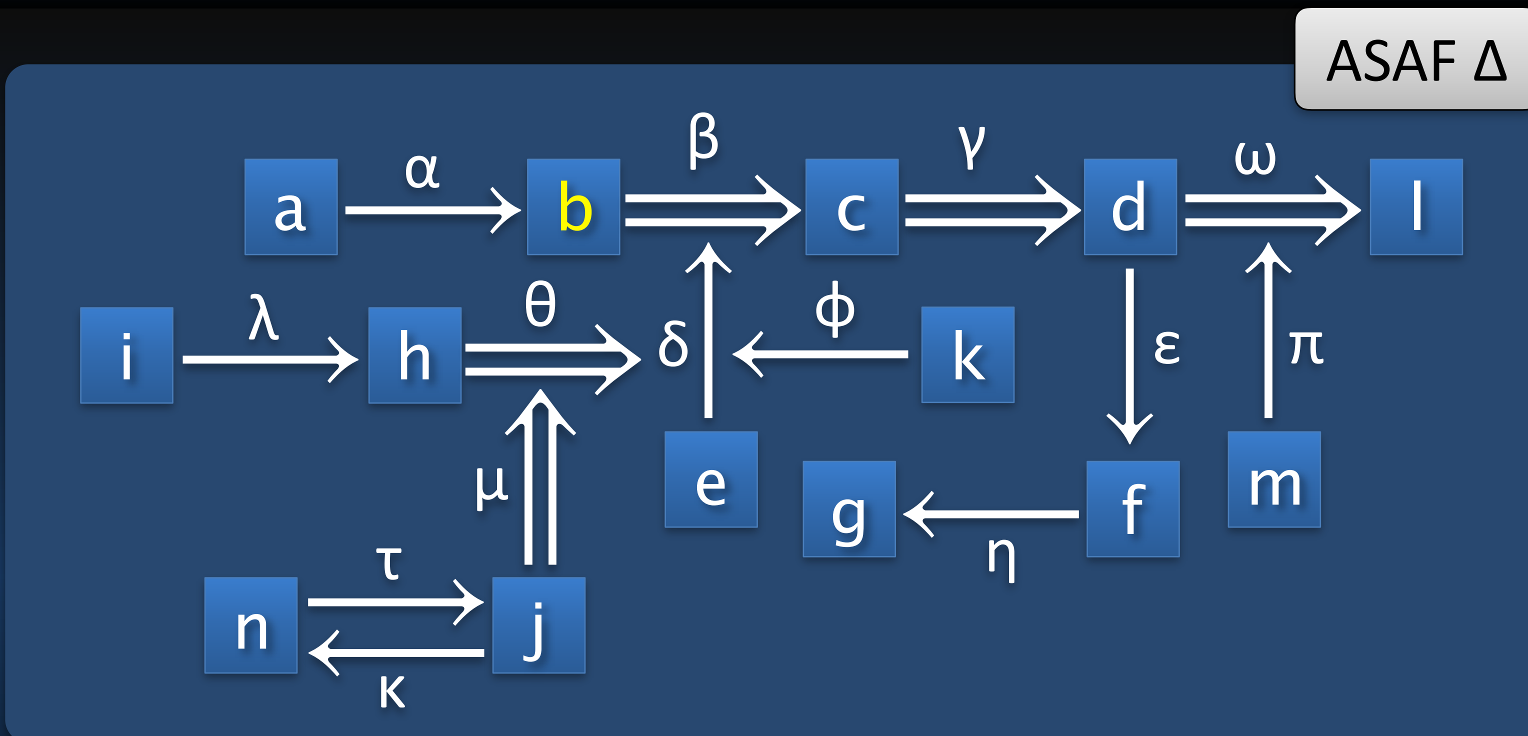


- b is not acceptable w.r.t. ∅; and
- δ is not acceptable w.r.t. {κ}

ASAF - Acceptability Example



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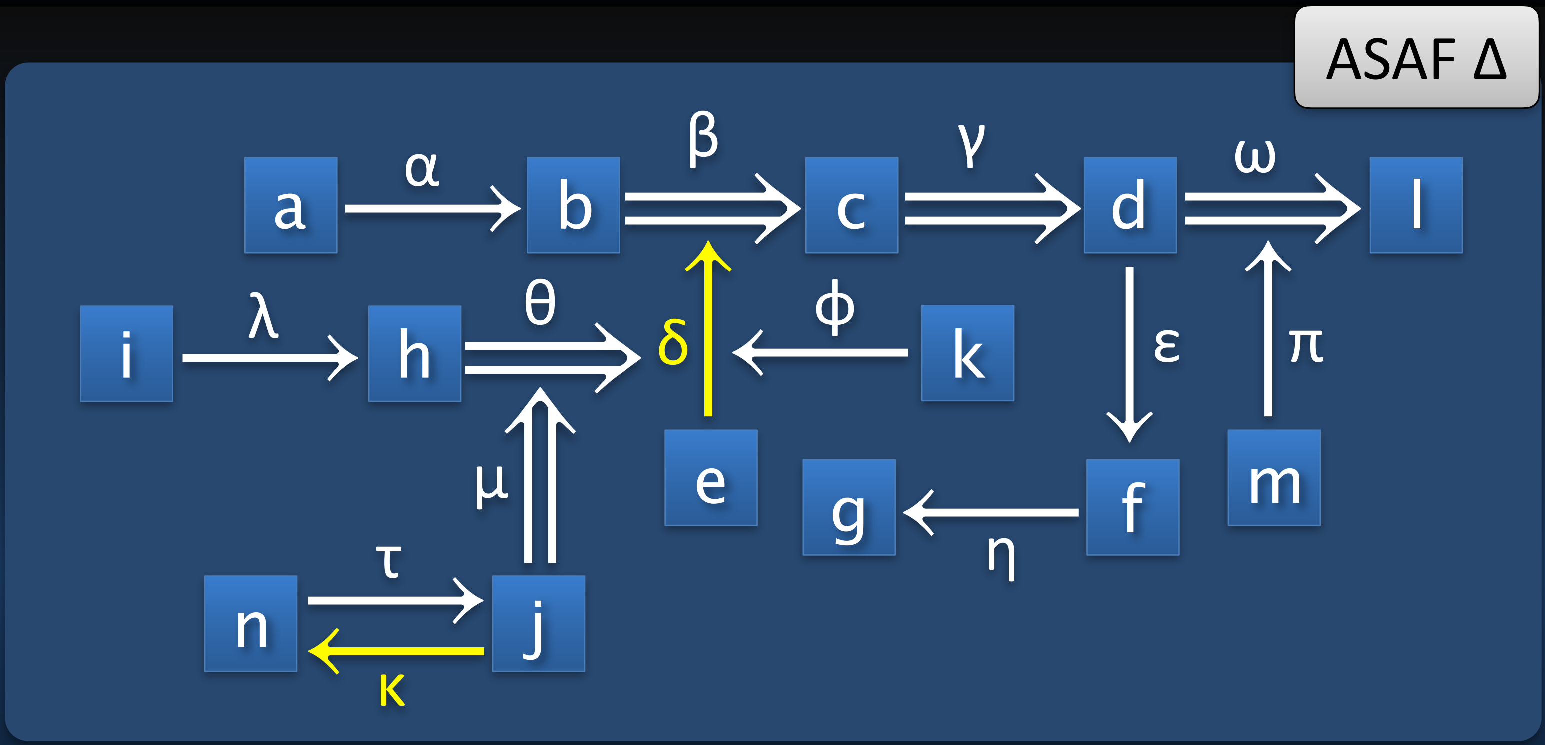

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ASAF - Admissibility

- The notion of admissibility characterizes some minimum requirements that a set of arguments, attacks and supports of the ASAF should satisfy in order to be collectively accepted.

- Let $\langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $\mathbf{S} \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$. We say $\mathbf{S}$ is admissible iff it is conflict-free and $\forall X \in \mathbf{S}$ it holds that X is acceptable w.r.t. $\mathbf{S}$.

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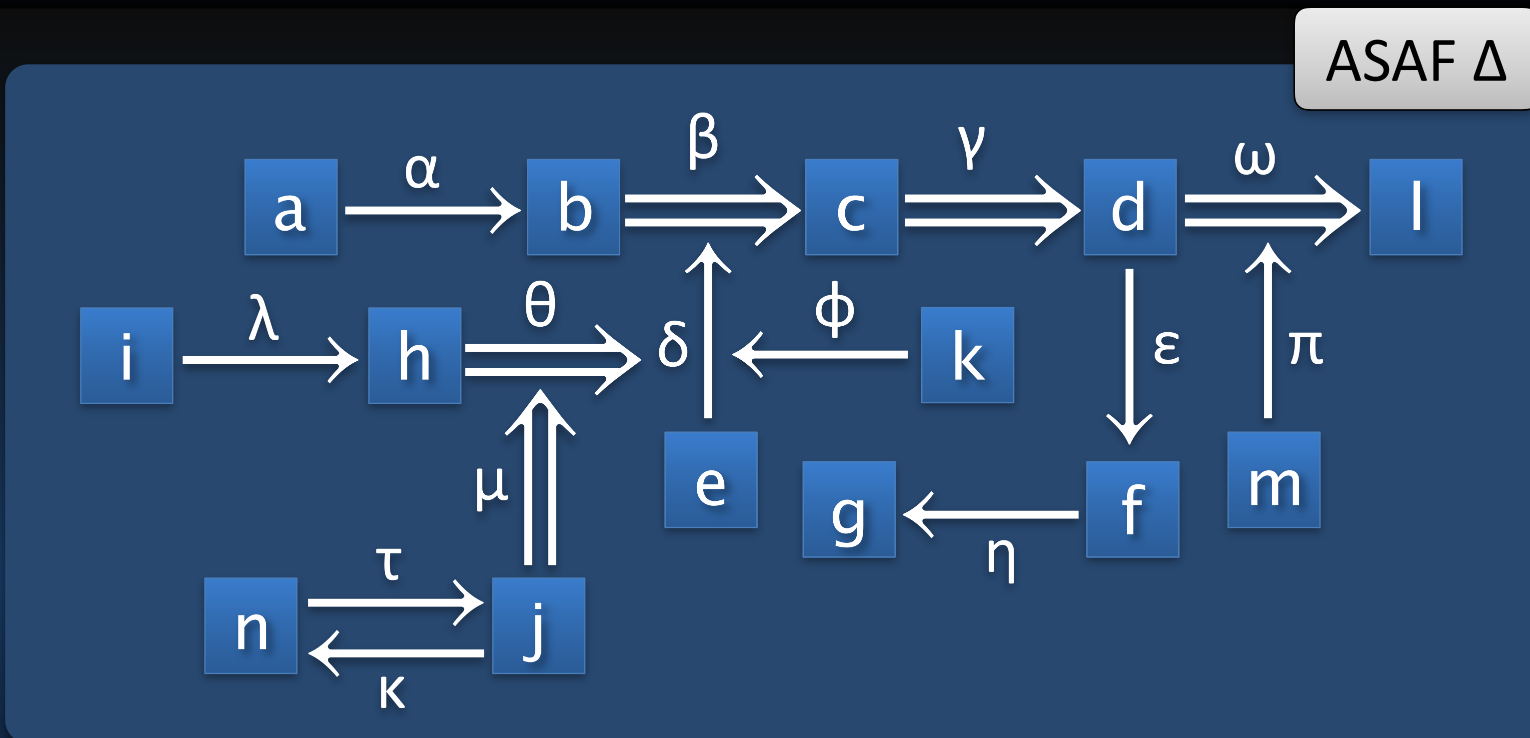
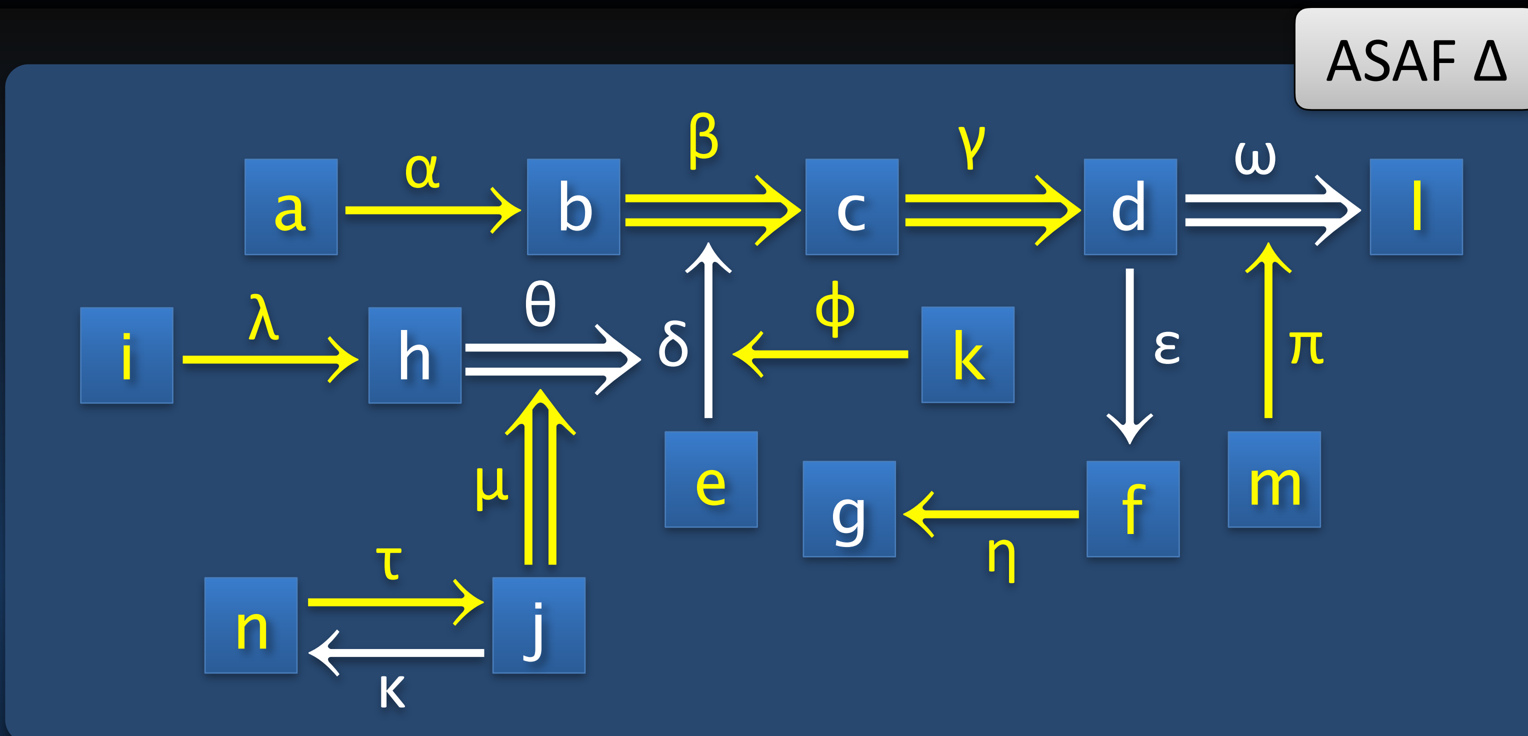


- Admissible Sets: {a,α,γ,m,π,l,i,λ,k,ϕ,β,f,η,e,μ,τ,n} and {α,β,γ,ϕ,f,m}.

ASAF - Admissibility Example



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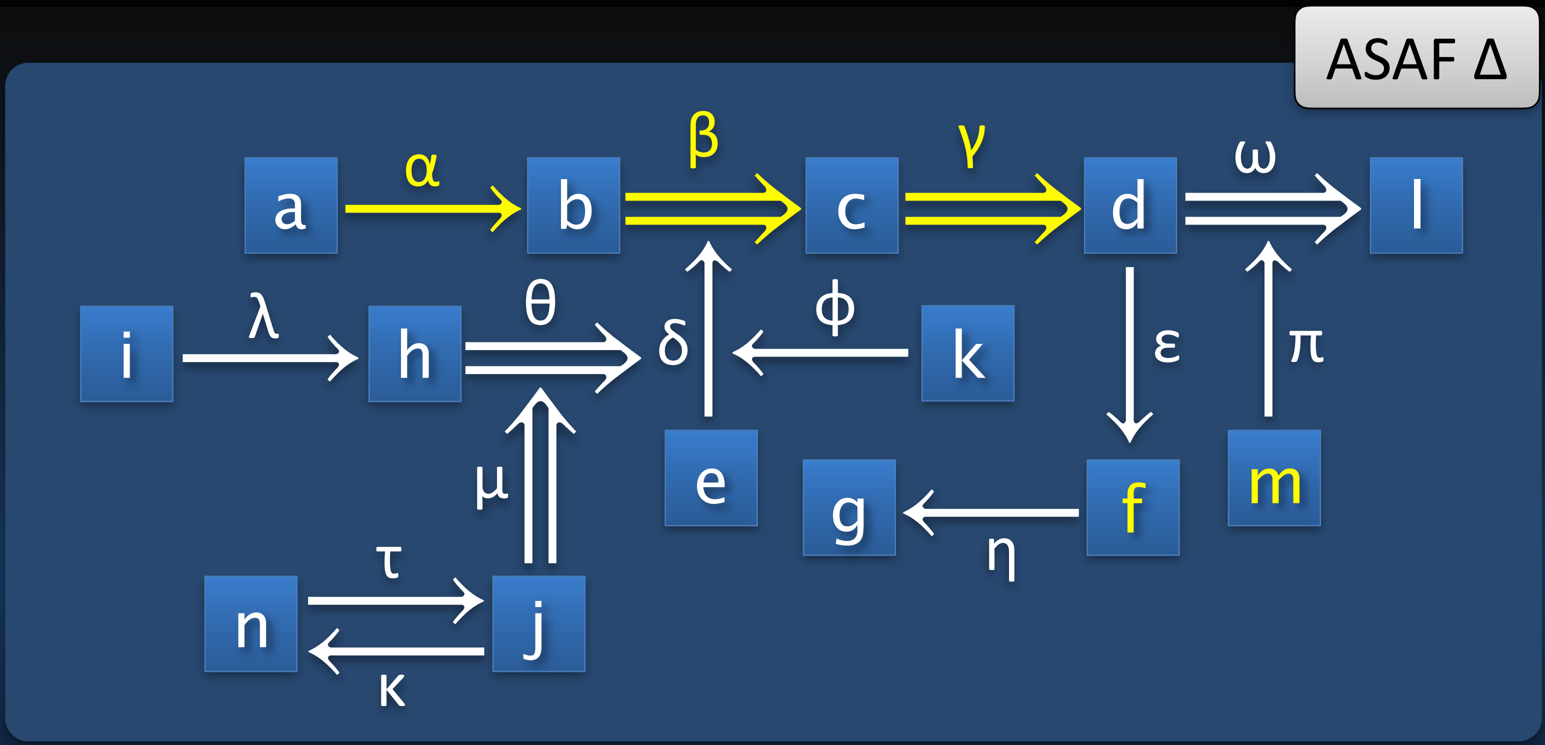

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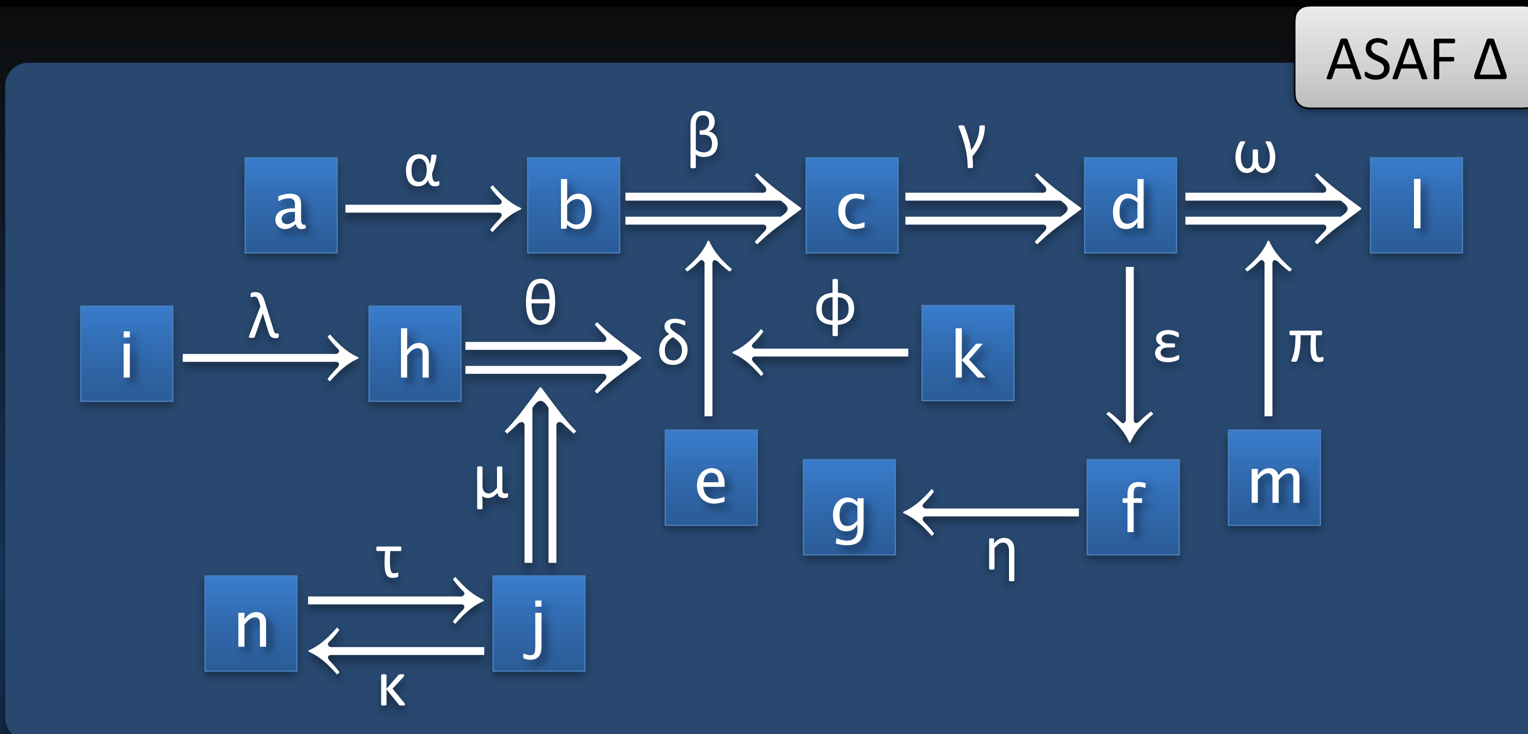
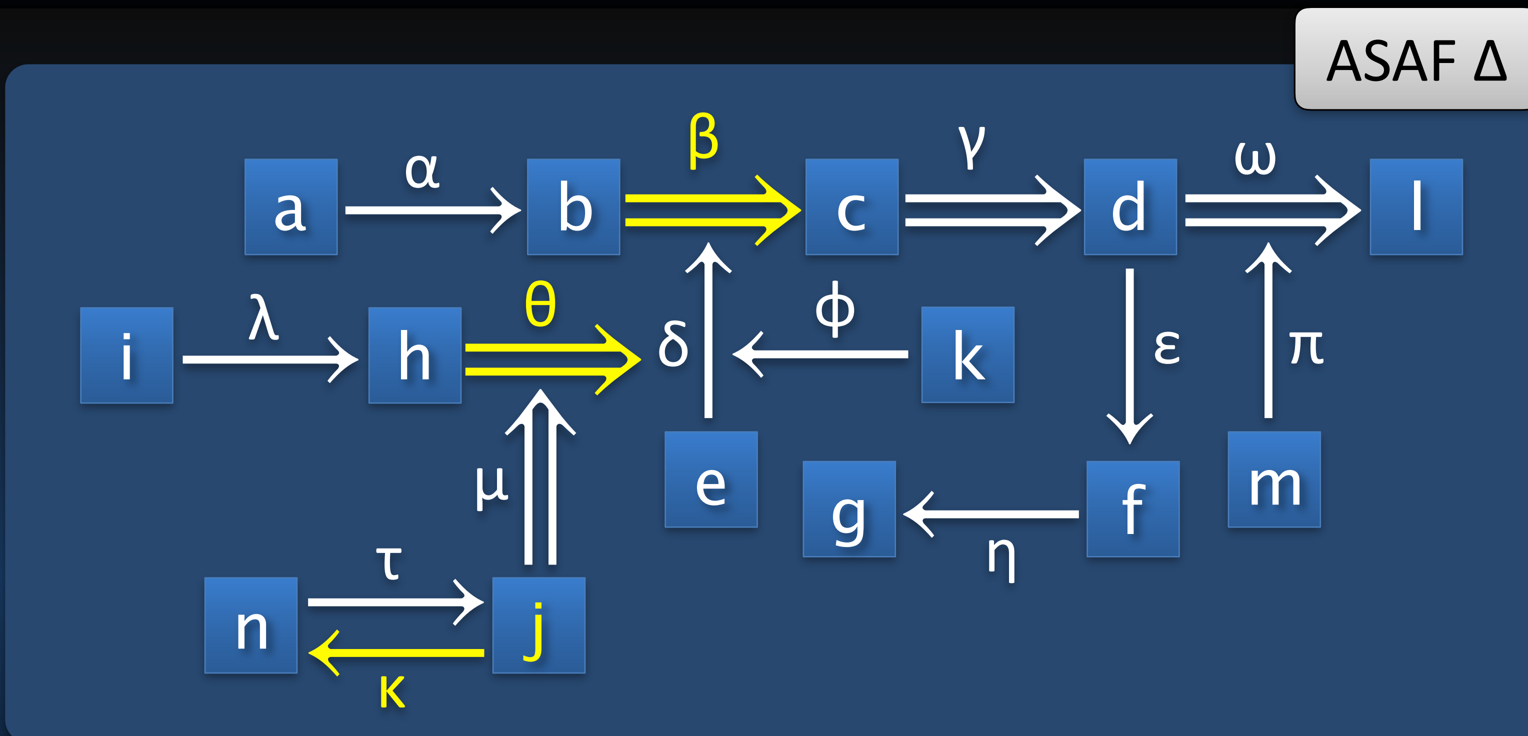


- Non-admissible Sets: {β,θ,j,κ} and {ε,g}.

ASAF - Admissibility Example



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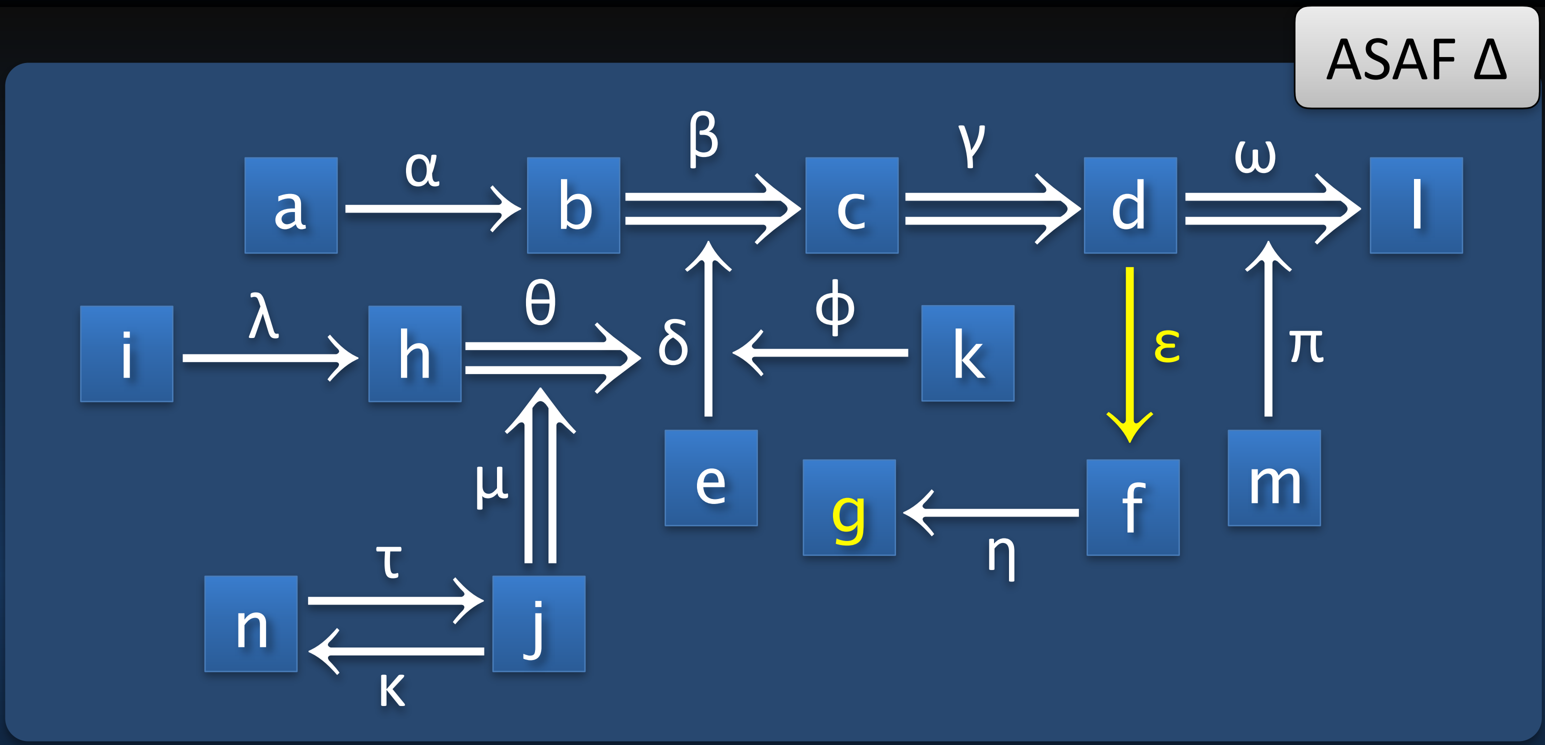


- Non-admissible Sets: {β,θ,j,κ} and {ε,g}.

Extensional Semantics of the ASAF

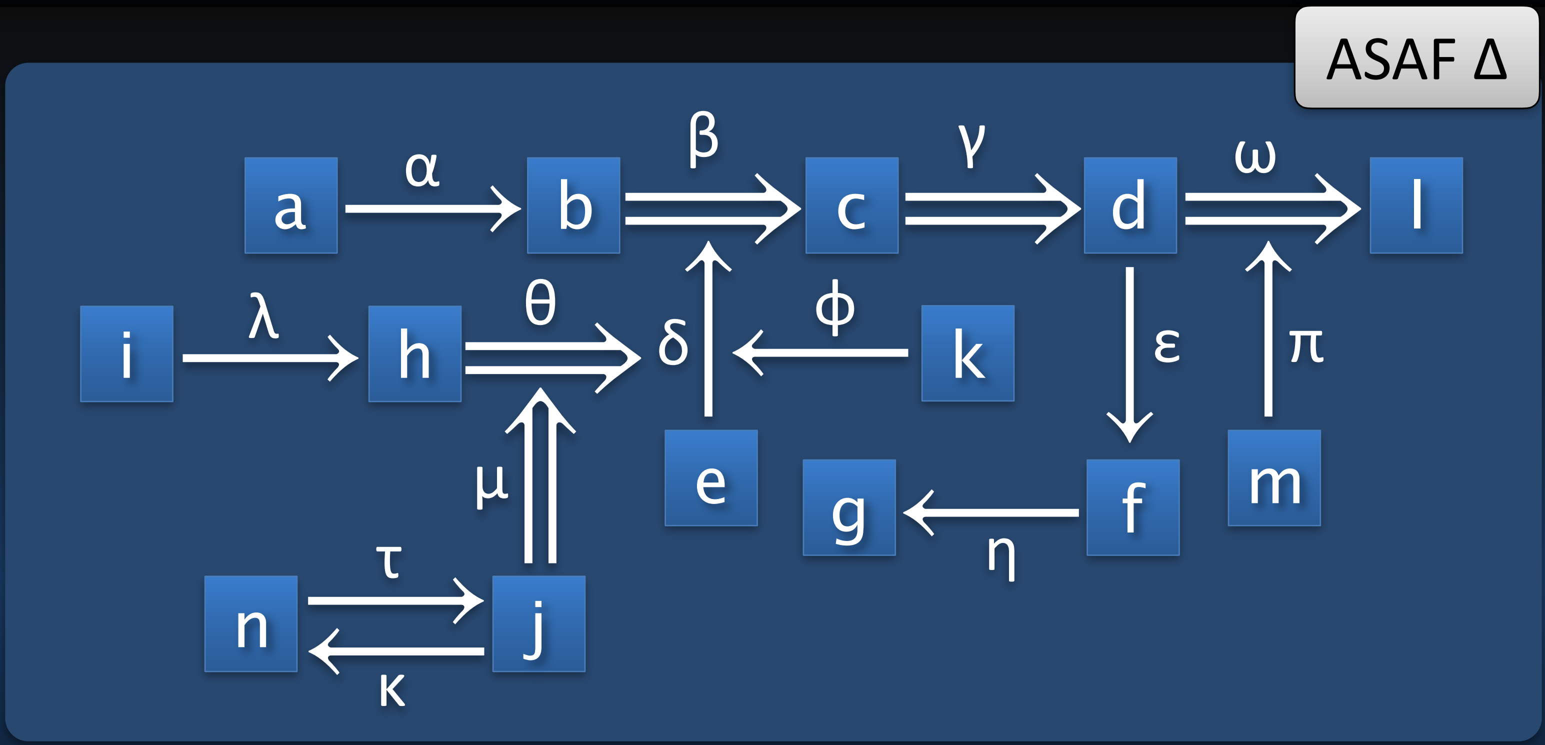
ASAF - Extensional Semantics

- Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $\mathbf{S} \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$:
  - $\mathbf{S}$ is a complete extension of Δ if it is admissible and $\forall X \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$: if X is acceptable w.r.t. $\mathbf{S}$, then $X \in \mathbf{S}$.
  - $\mathbf{S}$ is a preferred extension of Δ if it is a maximal (w.r.t. $\subseteq$) admissible set of Δ.
  - $\mathbf{S}$ is a stable extension of Δ if it is conflict-free and $\forall X \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S}) \setminus \mathbf{S}$: $\exists \alpha \in \mathbf{S}$, $\exists \mathbf{S}' \subseteq \mathbf{S}$ s.t. α u-def X or α c-def X given $\mathbf{S}'$.
  - $\mathbf{S}$ is the grounded extension of Δ if it is the smallest (w.r.t. $\subseteq$) complete extension of Δ.

ASAF - Extensional Semantics

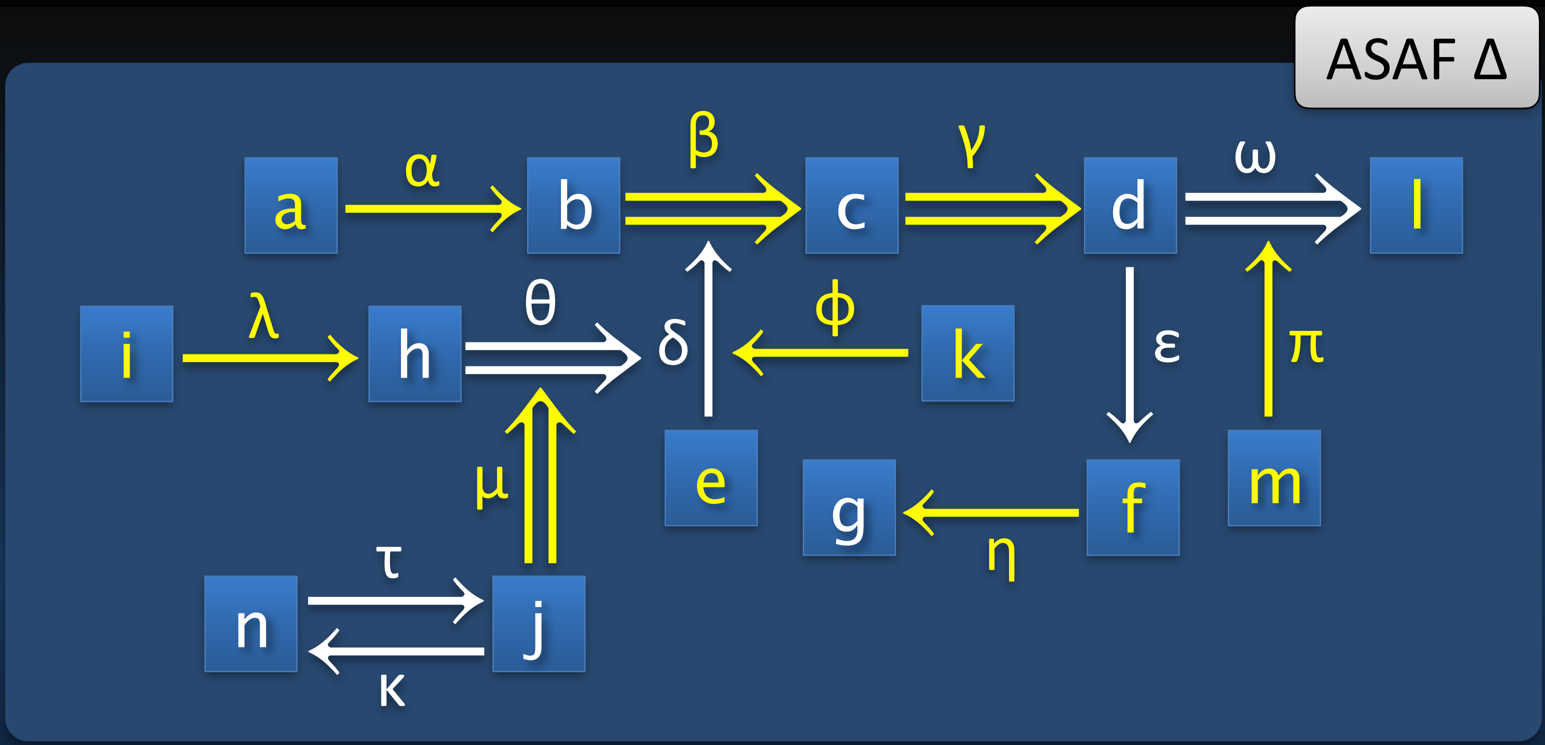
- Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF and $\mathbf{S} \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$:
  - **S** is a complete extension of Δ if it is admissible and $\forall X \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$: if X is acceptable w.r.t. **S**, then $X \in \mathbf{S}$.
  - **S** is a preferred extension of Δ if it is a maximal (w.r.t. ⊆) admissible set of Δ.
  - **S** is a stable extension of Δ if it is conflict-free and $\forall X \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S}) \setminus \mathbf{S}$: $\exists \alpha \in \mathbf{S}$, $\exists \mathbf{S}' \subseteq \mathbf{S}$ s.t. α u-def X or α c-def X given **S'**.
  - **S** is the grounded extension of Δ if it is the smallest (w.r.t. ⊆) complete extension of Δ.

ASAF - Extensions Example



- Grounded Extension: G = {a,α,γ,m,π,l,i,λ,k,ϕ,β,f,η,e,μ}.

ASAF - Extensions Example



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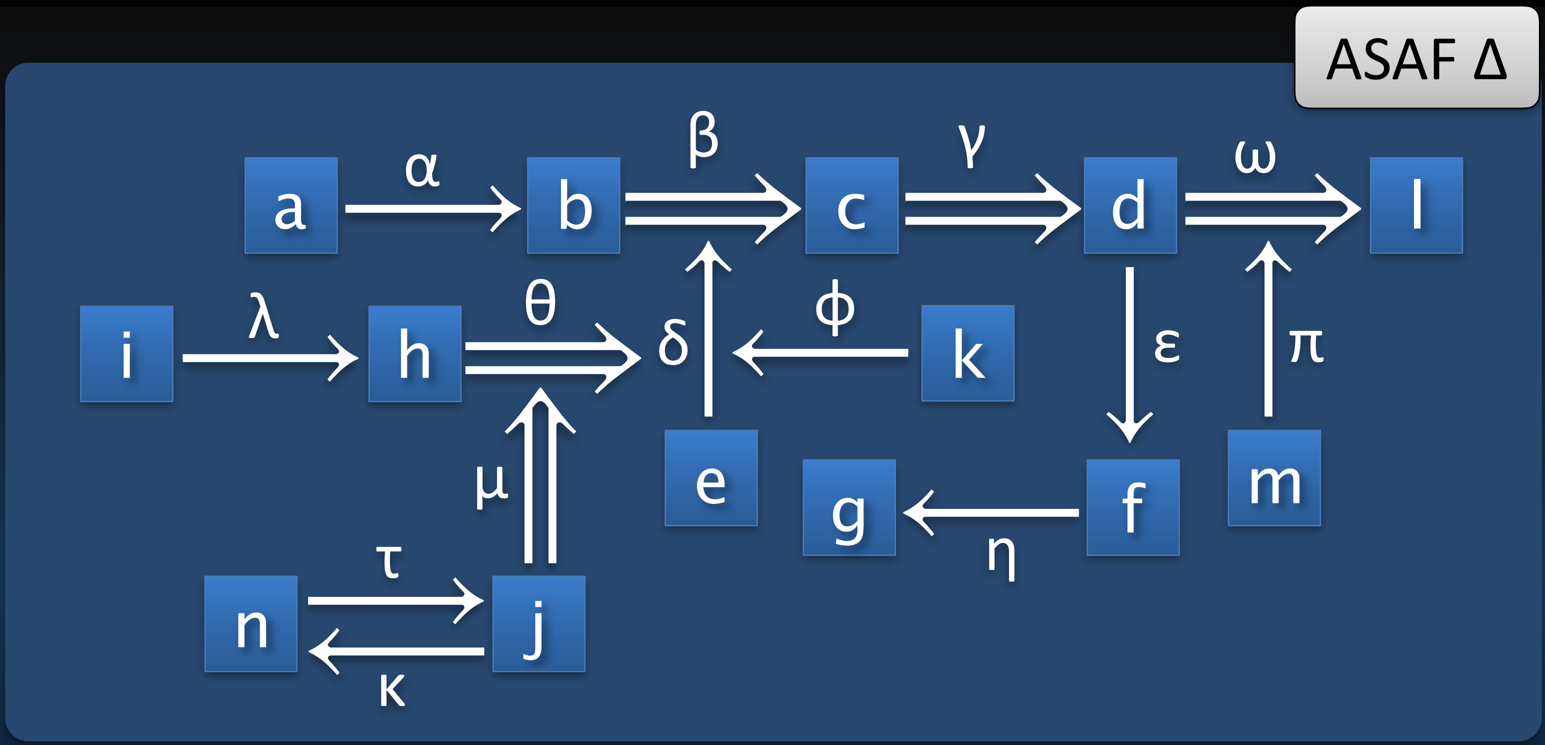
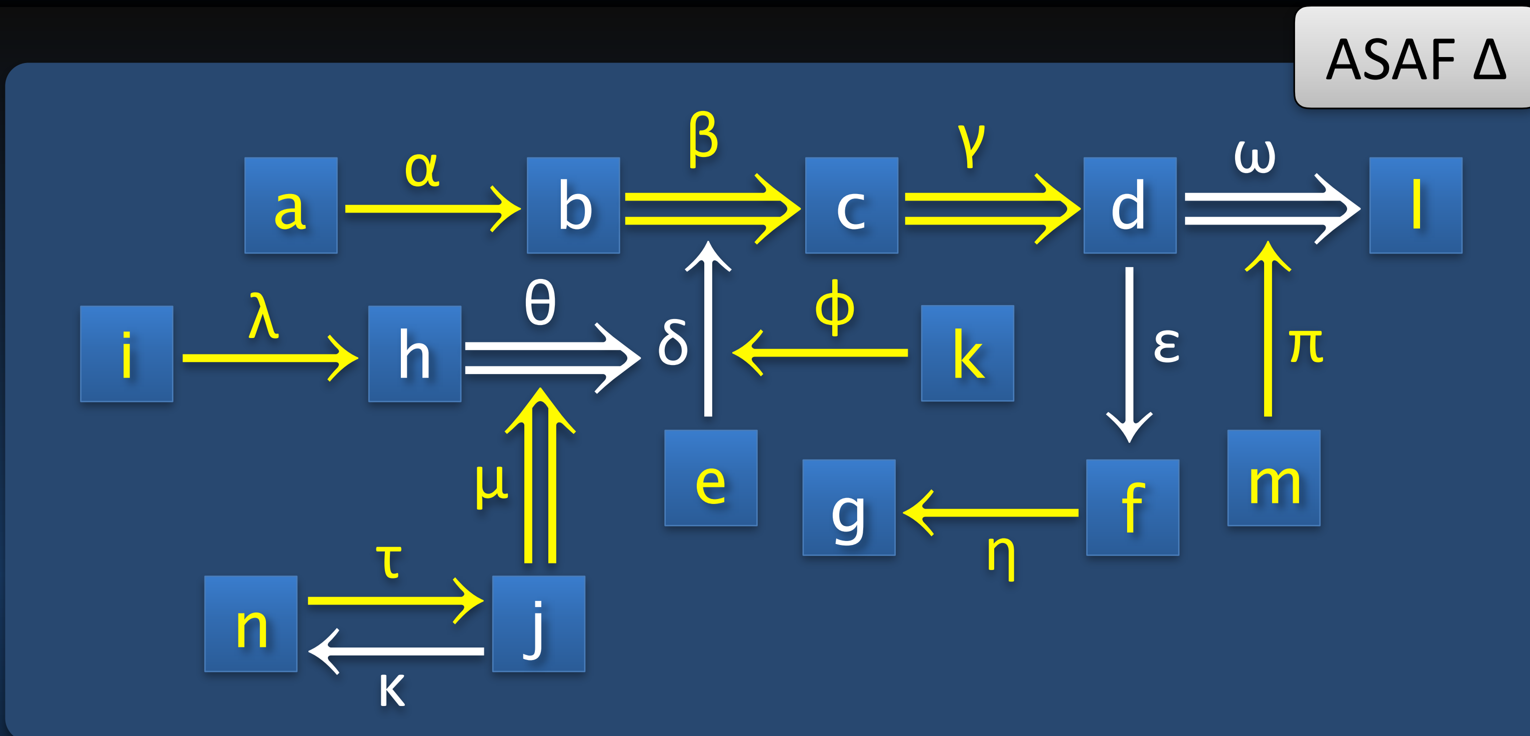


- Preferred Extensions: $P_1 = G \cup \{\tau, n\}$ and $P_2 = G \cup \{\kappa, j, \theta\}$.

ASAF - Extensions Example



- Preferred Extensions: **P_1= G** ∪ {τ,n} and P_2= G ∪ {κ,j,θ}.

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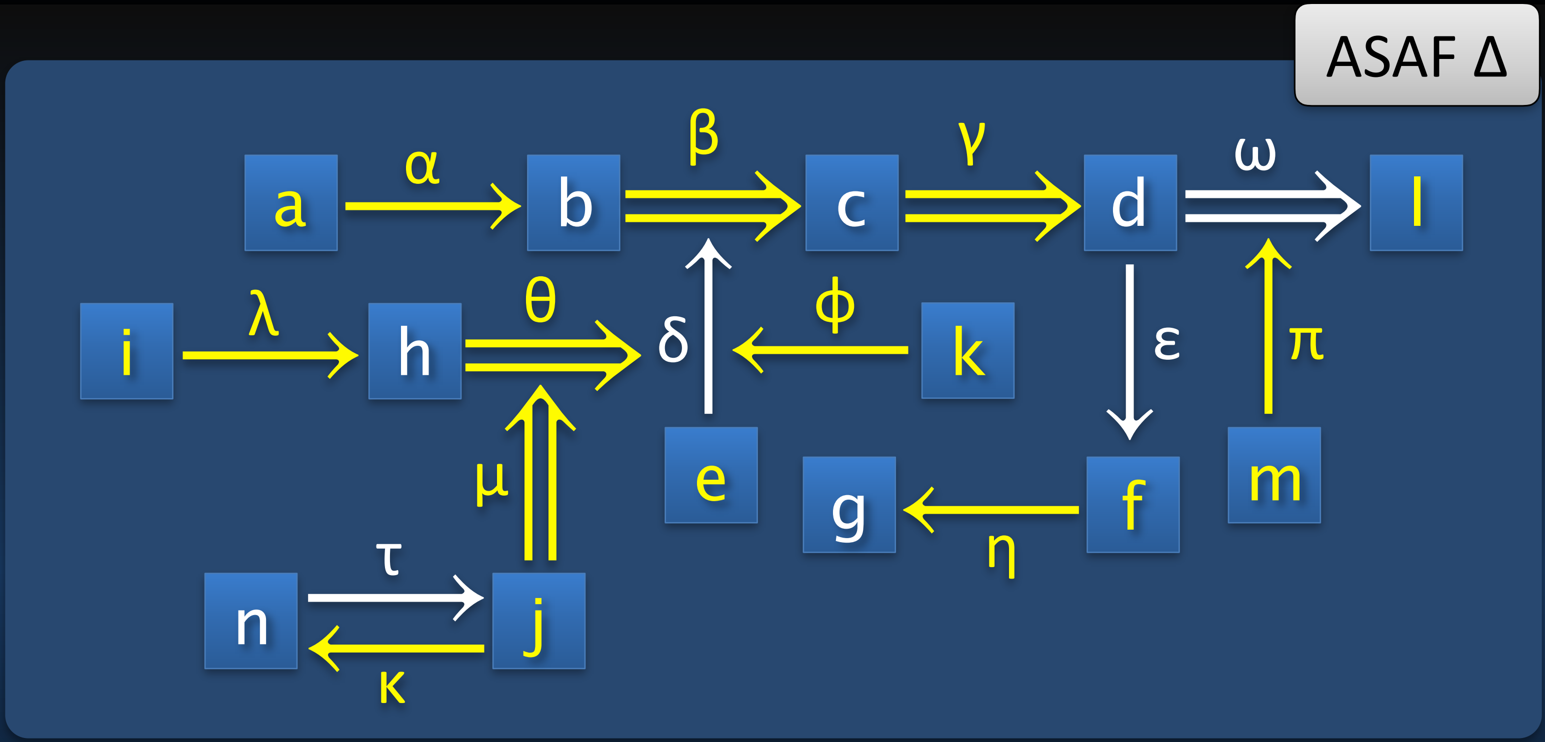

- Preferred Extensions: $\mathbf{P_1} = \mathbf{G} \cup \{\tau, n\}$ and $\mathbf{P_2} = \mathbf{G} \cup \{\kappa, j, \theta\}$.

Formal Results

ASAF - Formal Results

- Proposition: Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF, $\alpha \in \mathbb{R}$ and **S** $\subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$. If α is acceptable w.r.t. **S**, then src(α) is acceptable w.r.t. **S.**

- Proposition: Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF, **S** $\subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ a conflict-free set and $\alpha \in \mathbb{S}$ acceptable w.r.t. **S**. If trg(α) is acceptable w.r.t. **S**, then src(α) is acceptable w.r.t. **S**.
(Equivalently, if src(α) is not acceptable w.r.t. **S**, then trg(α) is not acceptable w.r.t. **S**).

ASAF - Formal Results

- Proposition: Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF, $X \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ and $\mathbf{S} \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$. If X is acceptable w.r.t. **S**, then $\forall \mathbf{S'} \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ s.t. $\mathbf{S} \subseteq \mathbf{S'}$: X is acceptable w.r.t. **S'**.

- Lemma: Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF, $\mathbf{S} \subseteq (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ an admissible set of Δ, and $X, Y \in (\mathbb{A} \cup \mathbb{R} \cup \mathbb{S})$ s.t. X and Y are acceptable w.r.t. **S**. Then, it holds that:
  (1) $\mathbf{S'} = \mathbf{S} \cup \{X\}$ is admissible; and
  (2) Y is acceptable w.r.t. **S'**.

ASAF - Formal Results

- Lemma: Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF. Every preferred extension of Δ is also a complete extension of Δ, but not vice-versa.

- Lemma: Let $\Delta = \langle \mathbb{A}, \mathbb{R}, \mathbb{S} \rangle$ be an ASAF. Every stable extension of Δ is also a preferred extension of Δ, but not vice-versa.

Thank You!

Questions?