A SPECIALIZED SET THEORETIC SEMANTICS FOR ACCEPTABILITY DYNAMICS OF ARGUMENTS

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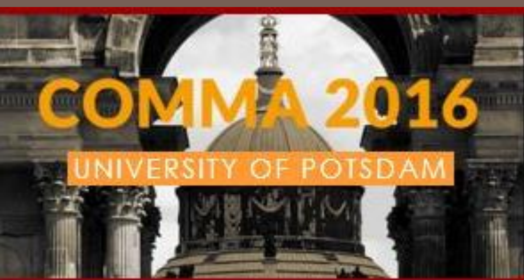


General Description

General Description: The Problem

We intend to deal with

- Reasoning about acceptability dynamics
- Through some Set Theoretic Semantics à la Tarski
- Handling the Dynamics of Arguments
- Allowing for reasoning about
  - the current state, and
  - the next one (through dynamics)

General Description

Abstract Argumentation

$$\mathcal{I}_S \mathrel{|\!\!\approx} a$$

- There is a way to alter the current framework st. the argument a ends up accepted by the semantics $\mathcal{S}$

General Description

Logic-based Argumentation

$$\mathcal{I}_S \mid\!\approx \vartheta$$

- There is a way to alter the current framework st. the formula ϑ ends up accepted by the semantics $\mathcal{S}$
  - Through some argument supporting ϑ

General Description

Logic-based Argumentation

$$\mathcal{I}_S \mid\!\approx \vartheta$$

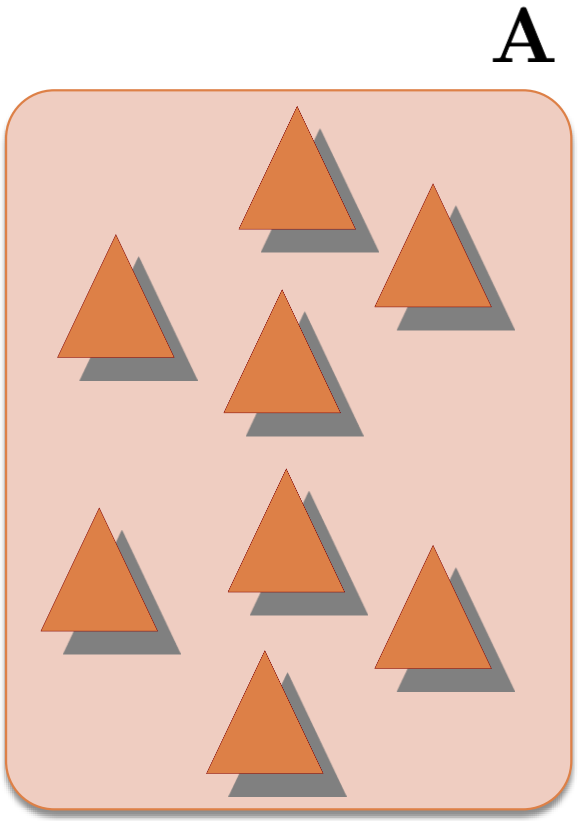
- There is a way to alter the current framework st. the formula ϑ ends up accepted by the semantics $\mathcal{S}$
  - Through some argument supporting ϑ

ContraSemantics is a model for answering that question, indicating how to evolve in that way

ContraSemantics

ContraSemantics

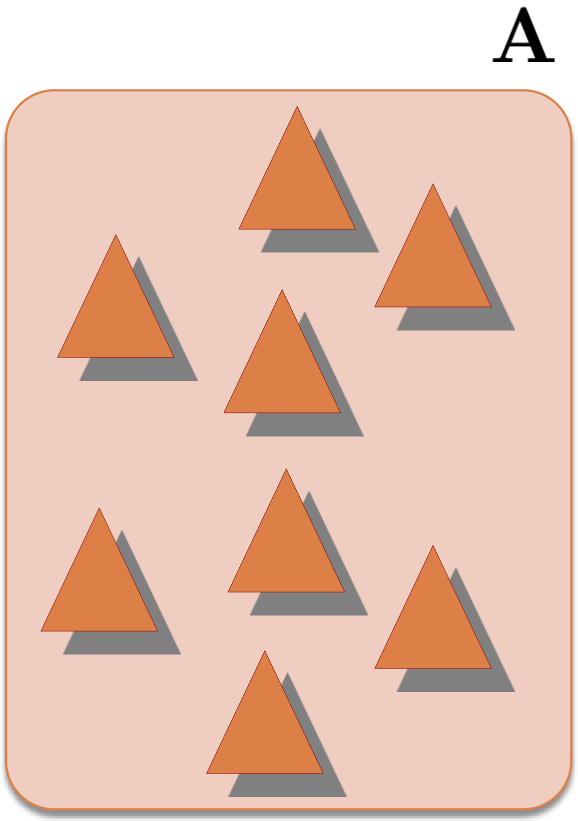
- An Argumentation Framework $\langle \mathbf{A}, \mathbf{R}_{\mathbf{A}} \rangle$



ContraSemantics

- An Argumentation Framework $\langle \mathbf{A}, \mathbf{R_A} \rangle$
- Dynamics Interpretation Structure

$$\mathcal{I}_{\mathcal{S}} = \langle \Delta^{\mathcal{I}}, \Gamma^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$$



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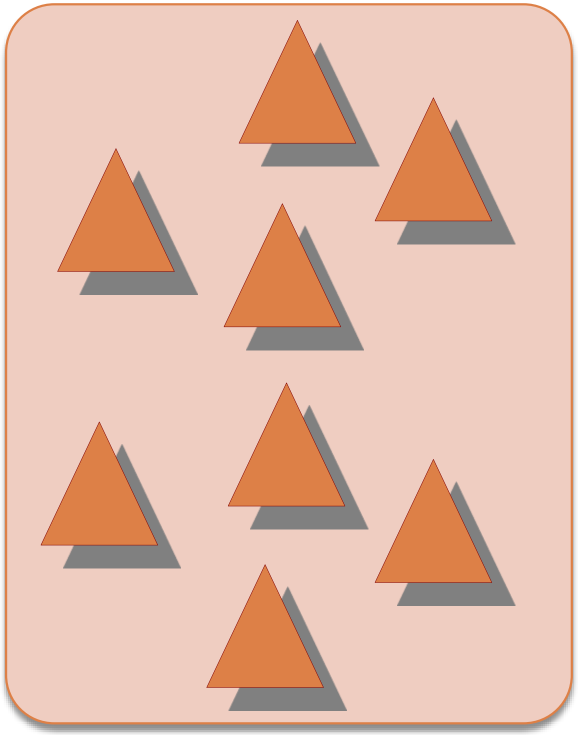
Argumentation Semantics

$\mathbf{A}$



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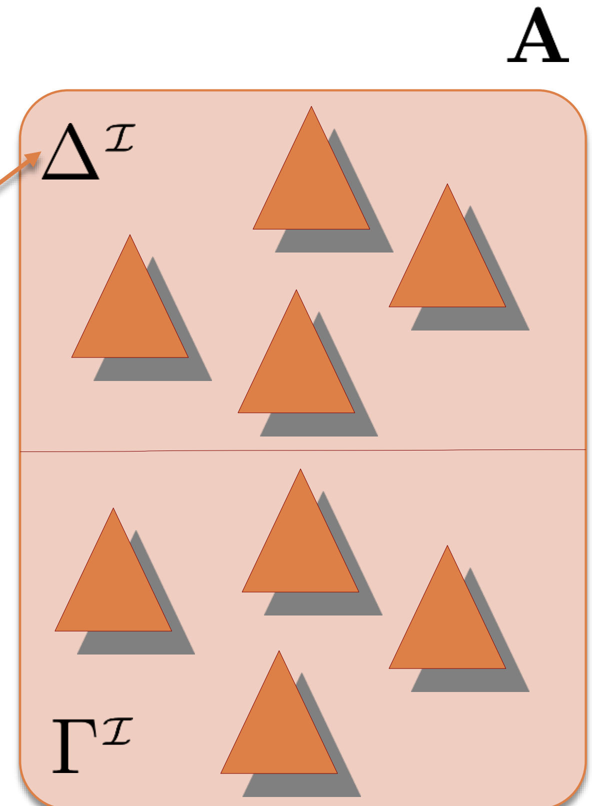
$$\mathcal{I}_{\mathcal{S}} = \langle \boxed{\Delta^{\mathcal{I}}}, \Gamma^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$$

Interpretation Domain



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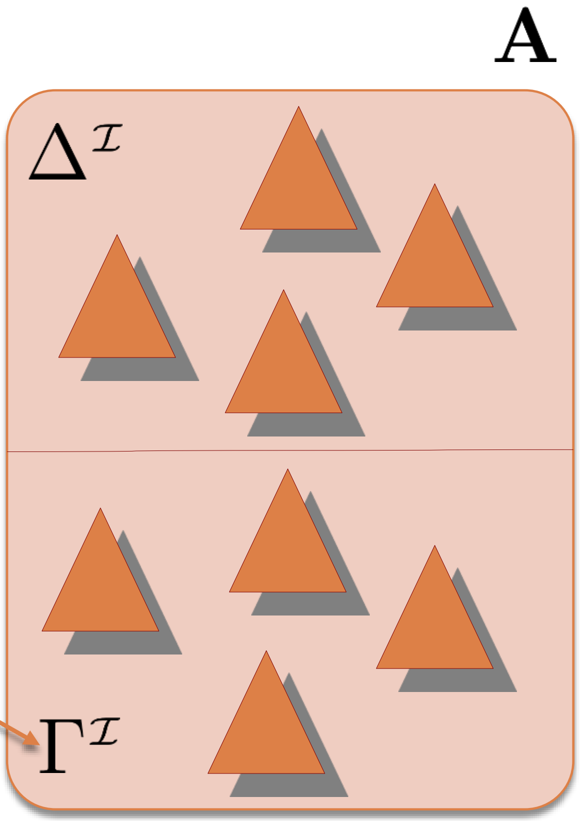
Interpretation Contra-Domain



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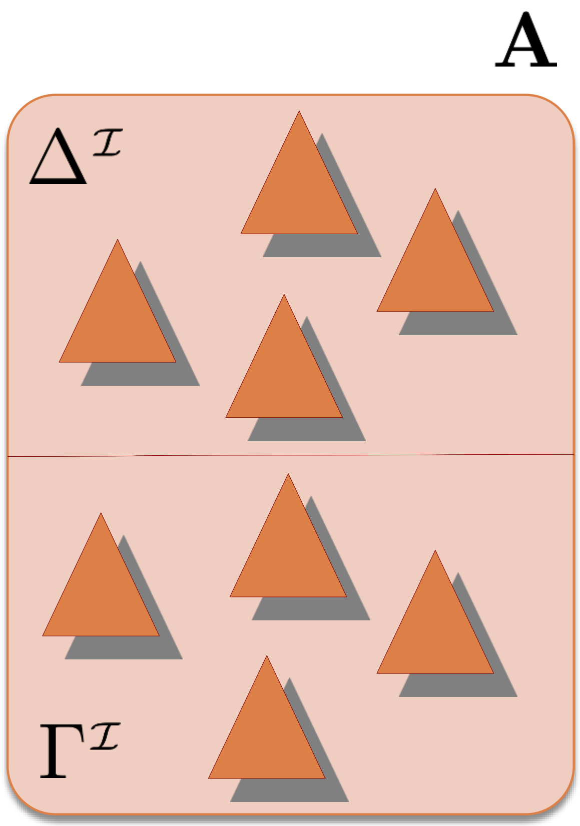
Interpretation Function



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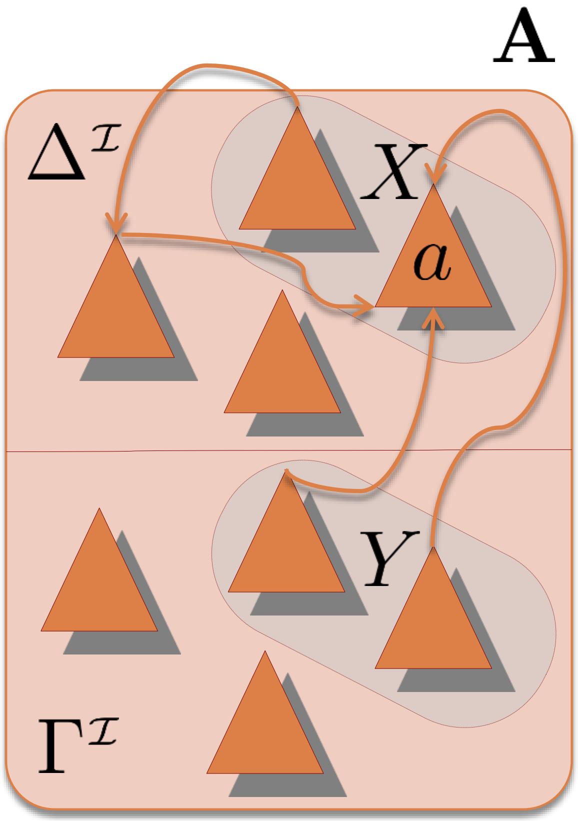
$$\mathcal{I}_S = \langle \Delta^{\mathcal{I}}, \Gamma^{\mathcal{I}}, \boxed{\cdot^{\mathcal{I}}} \rangle$$

Interpretation Function

$$\text{if } \mathcal{I}_S \mathrel{|}\!\approx \vartheta \text{ then}$$

$$(a, X, Y) \in \boxed{\vartheta^{\mathcal{I}}}$$



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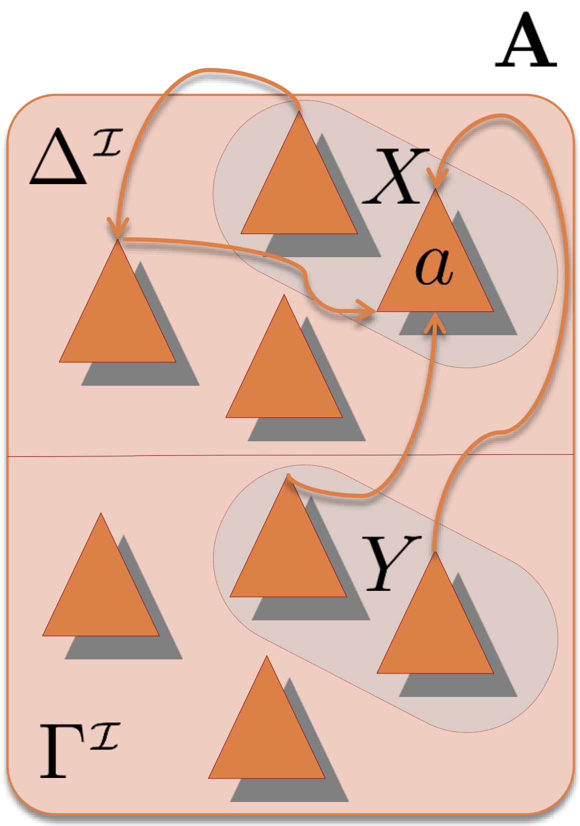
Interpretation Function

$$\text{if } \mathcal{I}_S \approx\!\!| \; \vartheta \text{ then}$$

$$(\boxed{a}, X, Y) \in \boxed{\vartheta^{\mathcal{I}}}$$

$$\boxed{\mathfrak{cl}(a) \models \vartheta}$$



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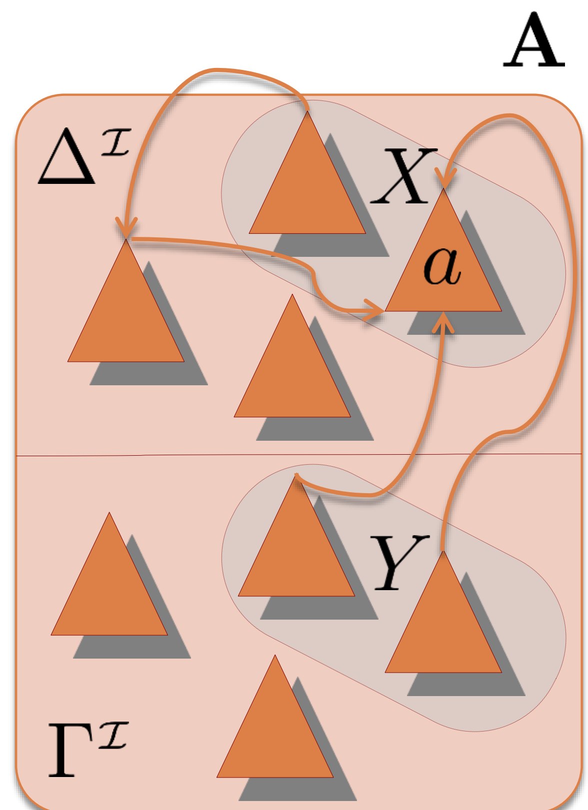
Interpretation Function

$$\text{if } \mathcal{I}_S \mathrel{|\!\approx} \vartheta \text{ then}$$

$$(a, \boxed{X}, Y) \in \boxed{\vartheta^{\mathcal{I}}}$$

$$\boxed{a\text{-}\mathtt{core}_S \text{ in } \langle \Delta^{\mathcal{I}}, \mathbf{R}_{\Delta^{\mathcal{I}}} \rangle}$$

$$\mathfrak{cl}(a) \models \vartheta$$



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$$\mathcal{I}_S = \langle \Delta^{\mathcal{I}}, \Gamma^{\mathcal{I}}, \boxed{\cdot^{\mathcal{I}}} \rangle$$

Interpretation Function

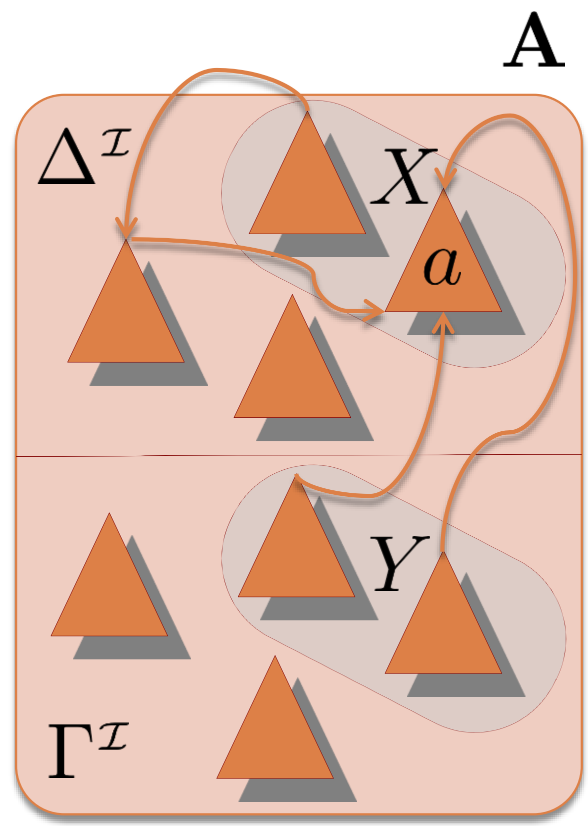
if $\mathcal{I}_S \mathrel{|\!\approx} \vartheta$ then

$$(a, X, \boxed{Y}) \in \boxed{\vartheta^{\mathcal{I}}}$$

$\boxed{a\text{-}\mathtt{remainder}_S \text{ in } \langle \mathbf{A}, \mathbf{R}_{\mathbf{A}} \rangle}$

a-$\mathtt{core}_S$ in $\langle \Delta^{\mathcal{I}}, \mathbf{R}_{\Delta^{\mathcal{I}}} \rangle$

$\mathfrak{cl}(a) \models \vartheta$



ContraSemantics

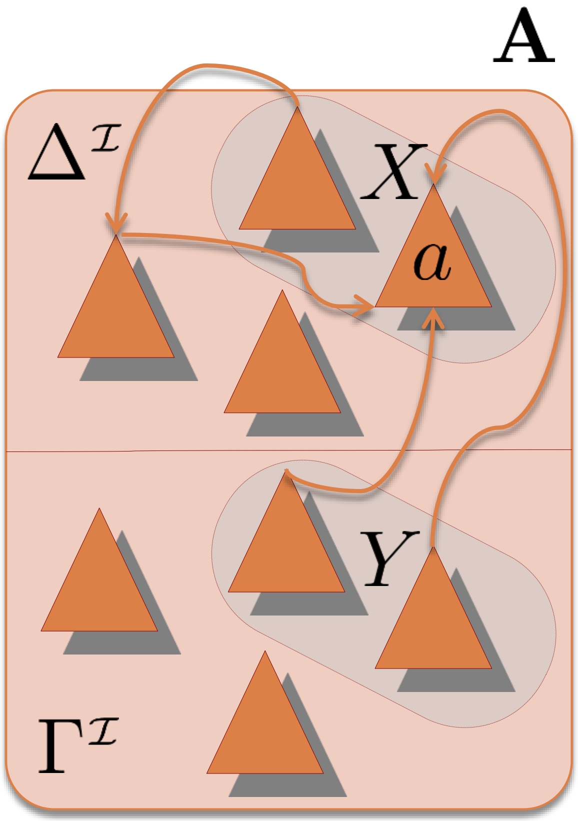
- An Argumentation Framework $\langle \mathbf{A}, \mathbf{R}_{\mathbf{A}} \rangle$
- Core and Reminder Sets

  a-$\texttt{core}_{\boxed{S}}$ in $\langle \Delta^{\mathcal{I}}, \mathbf{R}_{\Delta^{\mathcal{I}}} \rangle$

  a-$\texttt{remainder}_{\boxed{S}}$ in $\langle \mathbf{A}, \mathbf{R}_{\mathbf{A}} \rangle$

  - Based on

Argumentation Semantics



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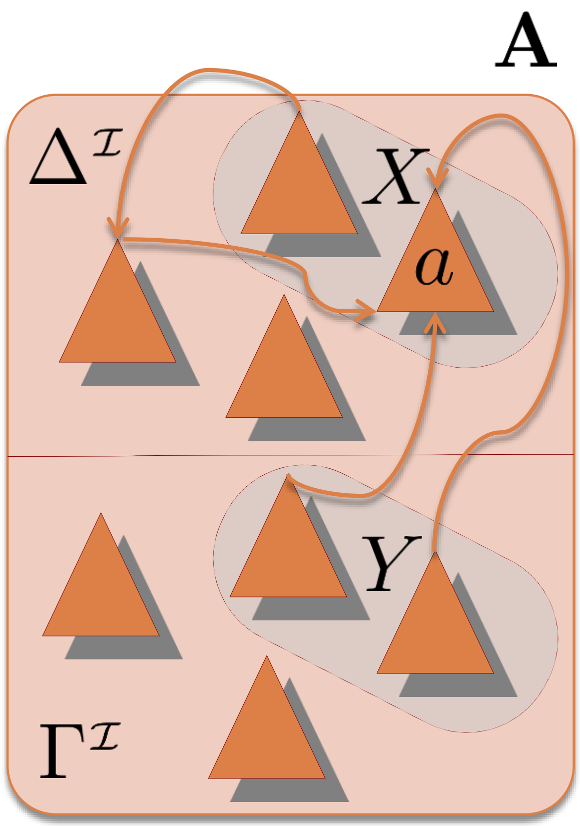
$$a\text{-}\mathtt{core}_{\mathcal{S}} \text{ in } \langle \Delta^{\mathcal{I}}, \mathbf{R}_{\Delta^{\mathcal{I}}} \rangle$$
$$a\text{-}\mathtt{remainder}_{\mathcal{S}} \text{ in } \langle \mathbf{A}, \mathbf{R}_{\mathbf{A}} \rangle$$

Core is a minimal set for ensuring the acceptability of a given argument according to a given semantics.



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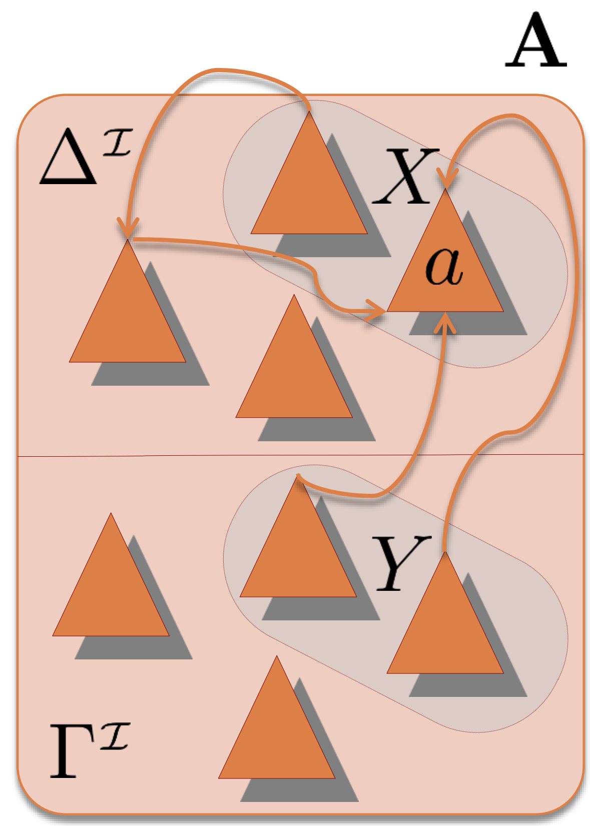
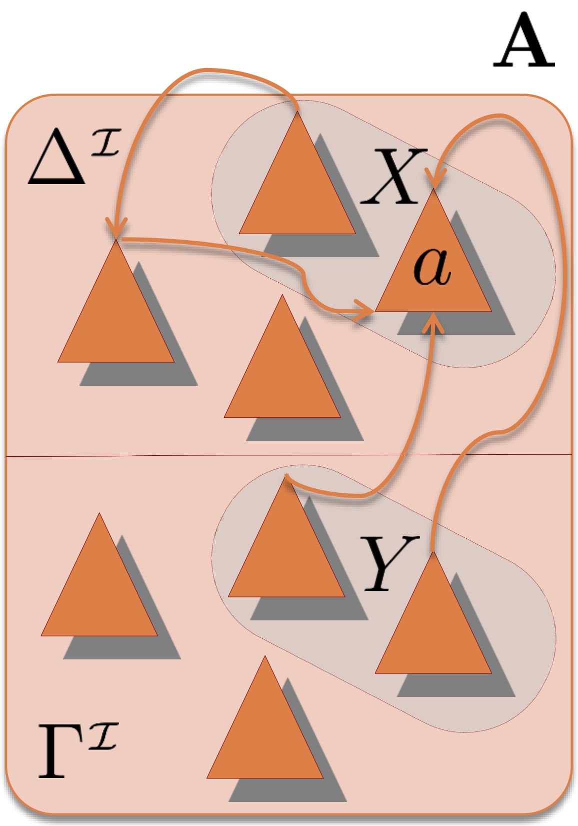
Core is a minimal set for ensuring the acceptability of a

Remainder is a minimal set interfering with the acceptability of a given argument according to a given semantics.



ContraSemantics

- An Argumentation Framework $\langle \mathbf{A}, \mathbf{R}_{\mathbf{A}} \rangle$
- Core and Reminder Sets
- if a is not accepted by $\mathcal{S}$ in $\langle \mathbf{A}, \mathbf{R}_{\mathbf{A}} \rangle$
  - There is no core for a
  - There is a remainder Y for a
  - There is a core X for a in $\langle \mathbf{A} \setminus Y, \mathbf{R}_{\mathbf{A} \setminus Y} \rangle$



ContraSemantics

- An Argumentation Framework $\langle \mathbf{A}, \mathbf{R}_{\mathbf{A}} \rangle$
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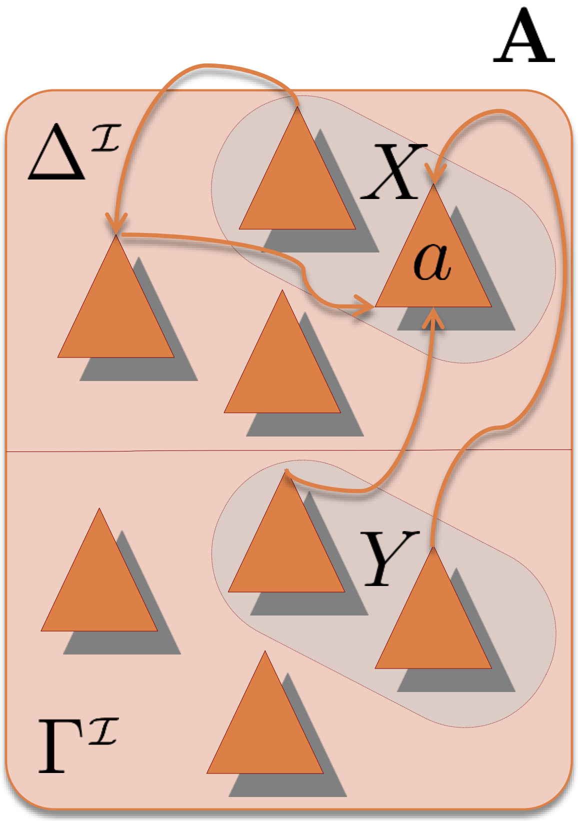
- If $X \subseteq \Delta^{\mathcal{I}}$ and $Y \subseteq \Gamma^{\mathcal{I}}$
  - a is accepted by $\mathcal{S}$ in $\langle \Delta^{\mathcal{I}}, \mathbf{R}_{\Delta^{\mathcal{I}}} \rangle$
  - and also in $\langle \mathbf{A} \setminus Y, \mathbf{R}_{\mathbf{A} \setminus Y} \rangle$



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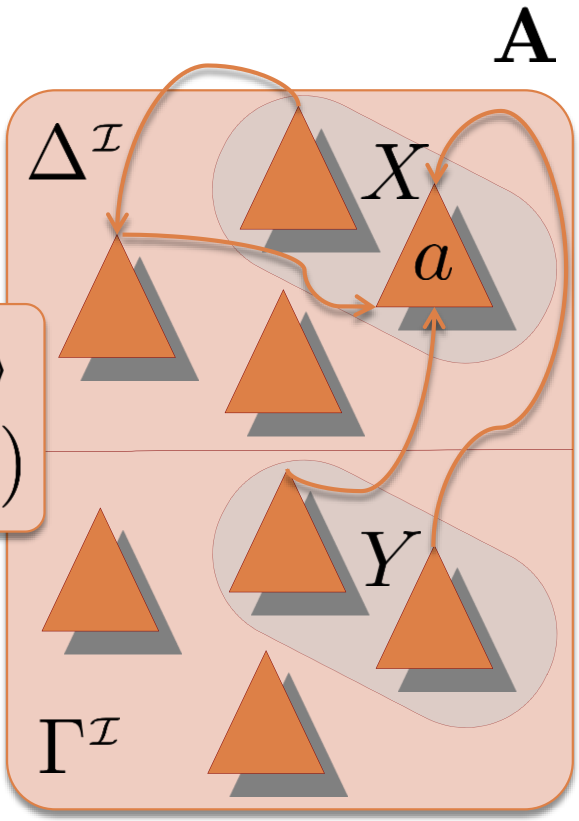
  - and also in $\langle \mathbf{A} \setminus Y, \mathbf{R}_{\mathbf{A} \setminus Y} \rangle$
- If $Y = \Gamma^{\mathcal{I}}$ then $\mathcal{I}_{\mathcal{S}} = \langle \Delta^{\mathcal{I}}, \Gamma^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$
  - is a minimal dynamic model for $\mathfrak{cl}(a)$



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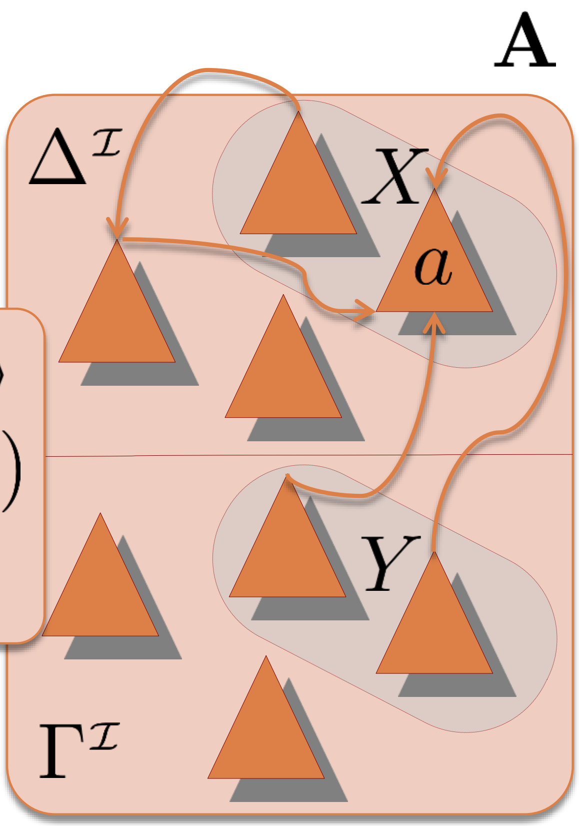

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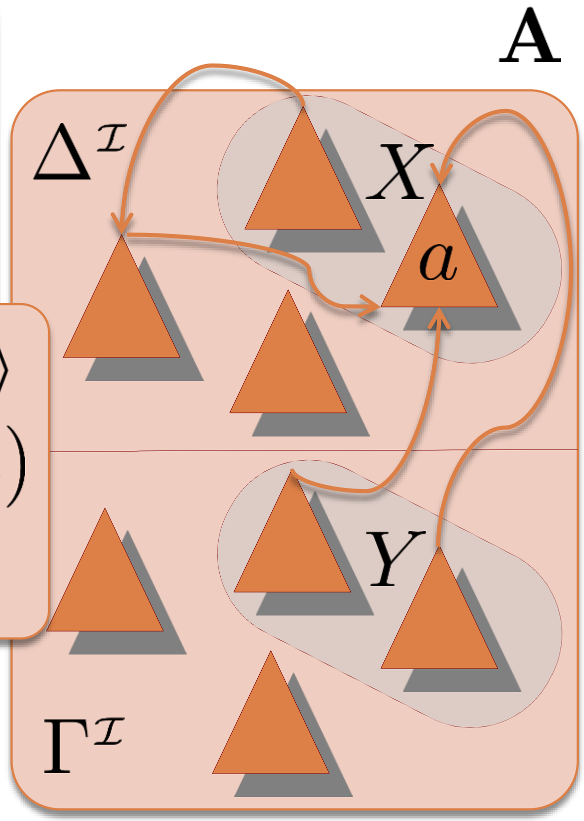
$(X, Y) \in a^{\mathcal{I}}$



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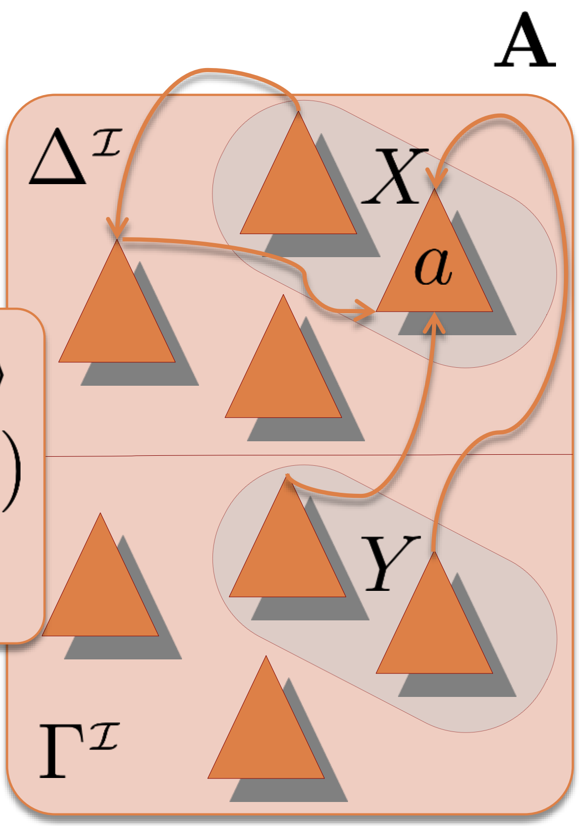
$a\text{-}\mathtt{core}_{\mathcal{S}}$ in $\langle \Delta^{\mathcal{I}}, \mathbf{R}_{\Delta^{\mathcal{I}}} \rangle$

$(X, Y) \in a^{\mathcal{I}}$



ContraSemantics

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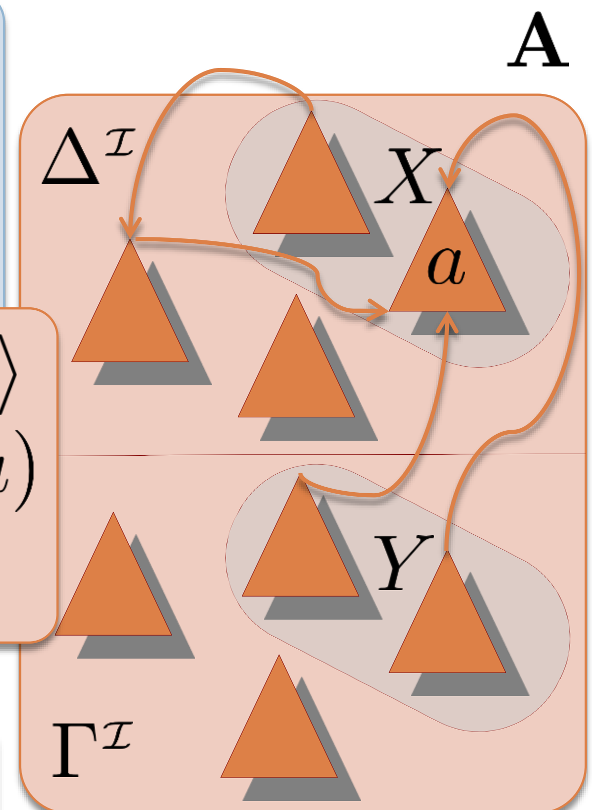
$(X, \boxed{Y}) \in a^{\mathcal{I}}$



ContraSemantics

Is it possible for a framework to evolve towards ensuring the acceptability of several arguments (conditions) ?

ContraSemantics

Is it possible for a framework to evolve

Is it possible to find an interpretation $\mathcal{I}_S$ st.

$$\mathcal{I}_S \overset{?}{\models\!\approx} \{a_1, \cdots, a_k\}$$

An Acceptance Revision Model

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- An Argumentation Framework $\tau = \langle \mathbf{A}, \mathbf{R}_{\mathbf{A}} \rangle$
- All minimal models of an argument $\mathcal{M}_{\mathcal{S}}(a, \tau)$
- The selection of the "best" minimal model

$$\gamma(\mathcal{M}_{\mathcal{S}}(a, \tau)) = \mathcal{I}_{\mathcal{S}} = \langle \Delta^{\mathcal{I}}, \Gamma^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$$

The Revision of the framework τ by an argument a is an evolved framework $\tau \circledast a = \langle \Delta^{\mathcal{I}}, \mathbf{R}_{\Delta^{\mathcal{I}}} \rangle$

An Acceptance Revision Model

- An Argumentation Framework $\tau = \langle \mathbf{A}, \mathbf{R_A} \rangle$
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Expand first for external arguments

An Acceptance Revision Model

- An Argumentation Framework $\tau = \langle \mathbf{A}, \mathbf{R_A} \rangle$

- The Expansion of the framework τ by an external argument a is an evolved framework
- $\tau + a = \langle \mathbf{A}', \mathbf{R}_{\mathbf{A}'} \rangle$ st. $\mathbf{A}' = \mathbb{C}(\mathbf{A}(\tau) \cup \{a\})$

An Acceptance Revision Model

- An Argumentation Framework $\tau = \langle \mathbf{A}, \mathbf{R}_{\mathbf{A}} \rangle$

- The Expansion of the framework τ by an external argument a is an evolved framework
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$a \notin \mathbf{A}(\tau)$

An Acceptance Revision Model

- An Argumentation Framework $\tau = \langle \mathbf{A}, \mathbf{R_A} \rangle$

- The Expansion of the framework τ by an external argument a is an evolved framework
- $\tau + a = \langle \mathbf{A}', \mathbf{R}_{\mathbf{A}'} \rangle$ st. $\mathbf{A}' = \boxed{\mathbb{C}(\mathbf{A}(\tau) \cup \{a\})}$

Argumentation Closure

Rationality

Rationality

(closure) if $\mathbf{A}(\tau) = \mathbb{C}(\mathbf{A}(\tau))$ then
$\mathbf{A}(\tau \circledast a) = \mathbb{C}(\mathbf{A}(\tau \circledast a))$

(success) a is $\mathcal{S}$-accepted in $\tau \circledast a$

(consistency) $\mathcal{A}_{\mathcal{S}}(\tau \circledast a)$ is conflict-free

(inclusion) $\mathbf{A}(\tau \circledast a) \subseteq \mathbf{A}(\tau + a)$

Rationality

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(consistency) $\boxed{\mathcal{A}_{\mathcal{S}}(\tau \circledast a)}$ is conflict-free

(inclusion) $\mathbf{A}(\tau \circledast a) \subseteq \mathbf{A}(\tau + a)$

Selected $\mathcal{S}$-extension

Rationality

(vacuity) If a is $\mathcal{S}$-accepted in $\tau + a$ then

$$\mathbf{A}(\tau + a) \subseteq \mathbf{A}(\tau \circledast a)$$

(core-retainment) If $b \in \mathbf{A}(\tau) \setminus \mathbf{A}(\tau \circledast a)$ then exists an $\mathtt{AF}$ τ' such that $\mathbf{A}(\tau') \subseteq \mathbf{A}(\tau)$ and a is $\mathcal{S}$-accepted in $\tau' + a$ but $\mathcal{S}$-rejected in $(\tau' + b) + a$

(uniformity) if $a \equiv b$ then

$$\mathbf{A}(\tau) \cap \mathbf{A}(\tau \circledast a) = \mathbf{A}(\tau) \cap \mathbf{A}(\tau \circledast b)$$

Rationality

Smooth Model Selection:

- Given $\mathcal{I}^a_S = \gamma(\mathcal{M}_S(a, \tau + a))$ and
  $\mathcal{I}^b_S = \gamma(\mathcal{M}_S(b, \tau + b))$
- If $a \equiv b$ then

$$\mathbb{C}((\Delta^{\mathcal{I}^a} \cap \Delta^{\mathcal{I}^b}) \cup \{a\}) = \Delta^{\mathcal{I}^a} \text{ and}$$
$$\mathbb{C}((\Delta^{\mathcal{I}^a} \cap \Delta^{\mathcal{I}^b}) \cup \{b\}) = \Delta^{\mathcal{I}^b}$$

Rationality

Smooth Model Selection:

- Given $\mathcal{I}_{\mathcal{S}}^{a} = \gamma(\mathcal{M}_{\mathcal{S}}(a, \tau + a))$ and
  $\mathcal{I}_{\mathcal{S}}^{b} = \gamma(\mathcal{M}_{\mathcal{S}}(b, \tau + b))$

- If $a \equiv b$ then

Argument Equivalence

$$\mathbb{C}((\Delta^{\mathcal{I}^a} \cap \Delta^{\mathcal{I}^b}) \cup \{a\}) = \Delta^{\mathcal{I}^a}$$ and
$$\mathbb{C}((\Delta^{\mathcal{I}^a} \cap \Delta^{\mathcal{I}^b}) \cup \{b\}) = \Delta^{\mathcal{I}^b}$$

Rationality

- Smooth Model Selection:
  - Given $\mathcal{I}_S^a = \gamma(\mathcal{M}_S($ ... and

    $\mathcal{I}_S^b = \gamma(\mathcal{M}_S$ ...

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$$\mathbb{C}((\Delta^{\mathcal{I}^a} \cap \Delta^{\mathcal{I}^b}) \cup \{a\}) = \Delta^{\mathcal{I}^a}$$ and

$$\mathbb{C}((\Delta^{\mathcal{I}^a} \cap \Delta^{\mathcal{I}^b}) \cup \{b\}) = \Delta^{\mathcal{I}^b}$$

There is a common "essential" set

Rationality

Representation Theorem:

- $\tau \circledast a$ is a smooth acceptance revision iff '$\circledast$' satisfies
  - closure,
  - success,
  - consistency,
  - inclusion,
  - vacuity,
  - core-retainment, and
  - uniformity.

Rationality

- Representation Theorem:
  - $\tau \circledast a$ is a smooth acceptance revision
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Smooth selection

- closure,
- success,
- consistency,
- inclusion,
- vacuity,
- core-retainment, and
- uniformity.

Conclusions

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- New theoretical structure conceived from scratch to deal with acceptability dynamics of arguments.
- The expected virtue of this theory is to ease the proposal and rationality analysis of new models of argumentative change.
- Simpler to show that the outcome of a “rational” change operator coincides with an interpretation model than showing the complete rationality through a representation theorem.
- If this hypothesis is true, the full rationality of new change operators could be achieved by means of the representation theorem here presented.

Conclusions

- The notions of core and remainder sets exceed the scope of the standard argumentation semantics.
- Their intuitions can be rationalized.
- Their constructions can be redefined for being applied over other kind of semantics and frameworks like dialectical argumentation.
- The reference to standard argumentation semantics in this work has been parametrized, thus allowing the modeling of marking criteria for trees of arguments (dialectical trees).

Thank you for your attention

Questions?